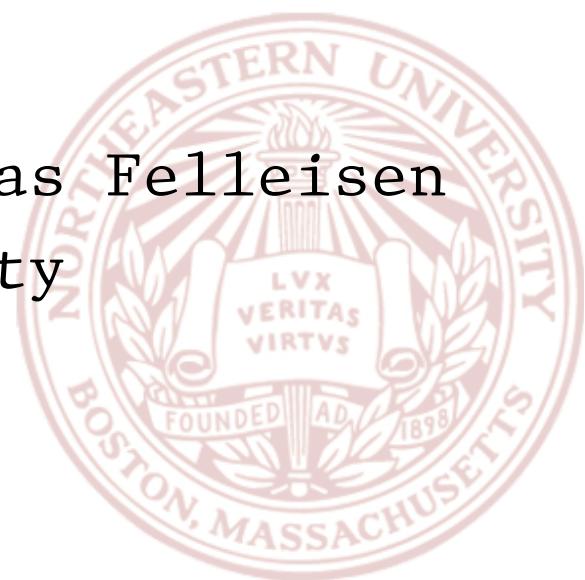


A Spectrum of Type Soundness and Performance

Ben Greenman & Matthias Felleisen
Northeastern University



Is type soundness all-or-nothing?

How does type soundness affect performance?

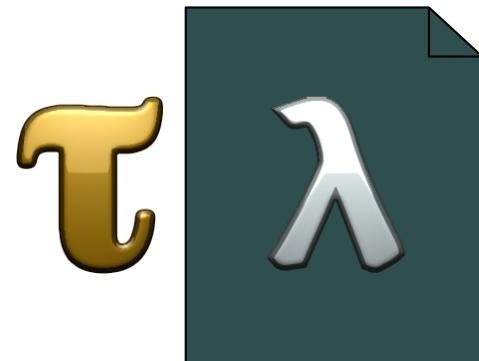
Migratory Typing

Migratory Typing



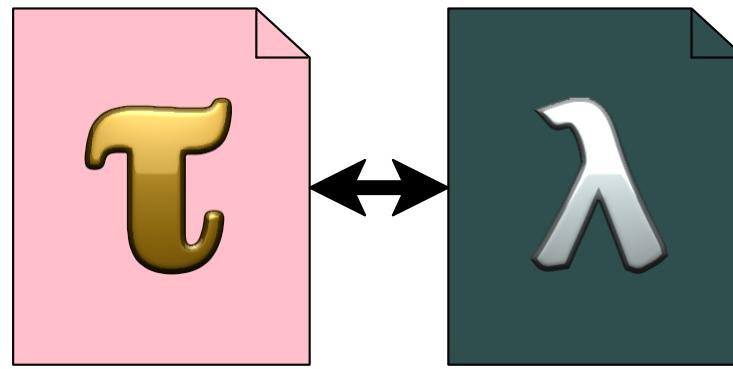
Begin with a un(i)typed language

Migratory Typing



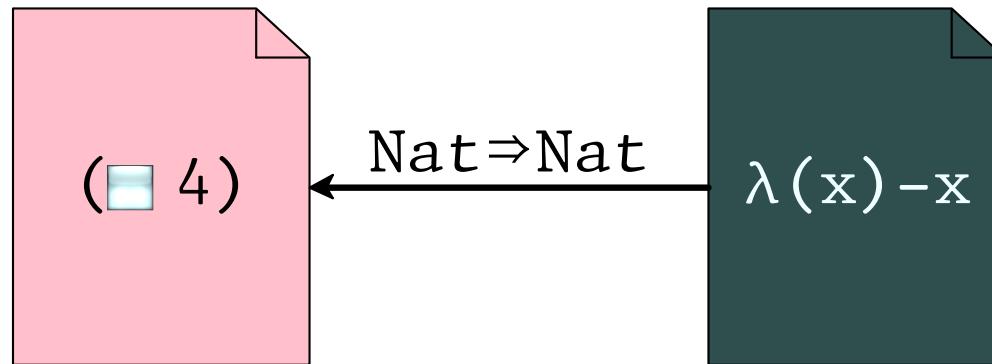
Design an *idiomatic* type system

Migratory Typing



Result: a mixed-typed language

Migratory Typing



Result: a mixed-typed language

Mixing Typed and Untyped Code

- migratory typing

gradual typing

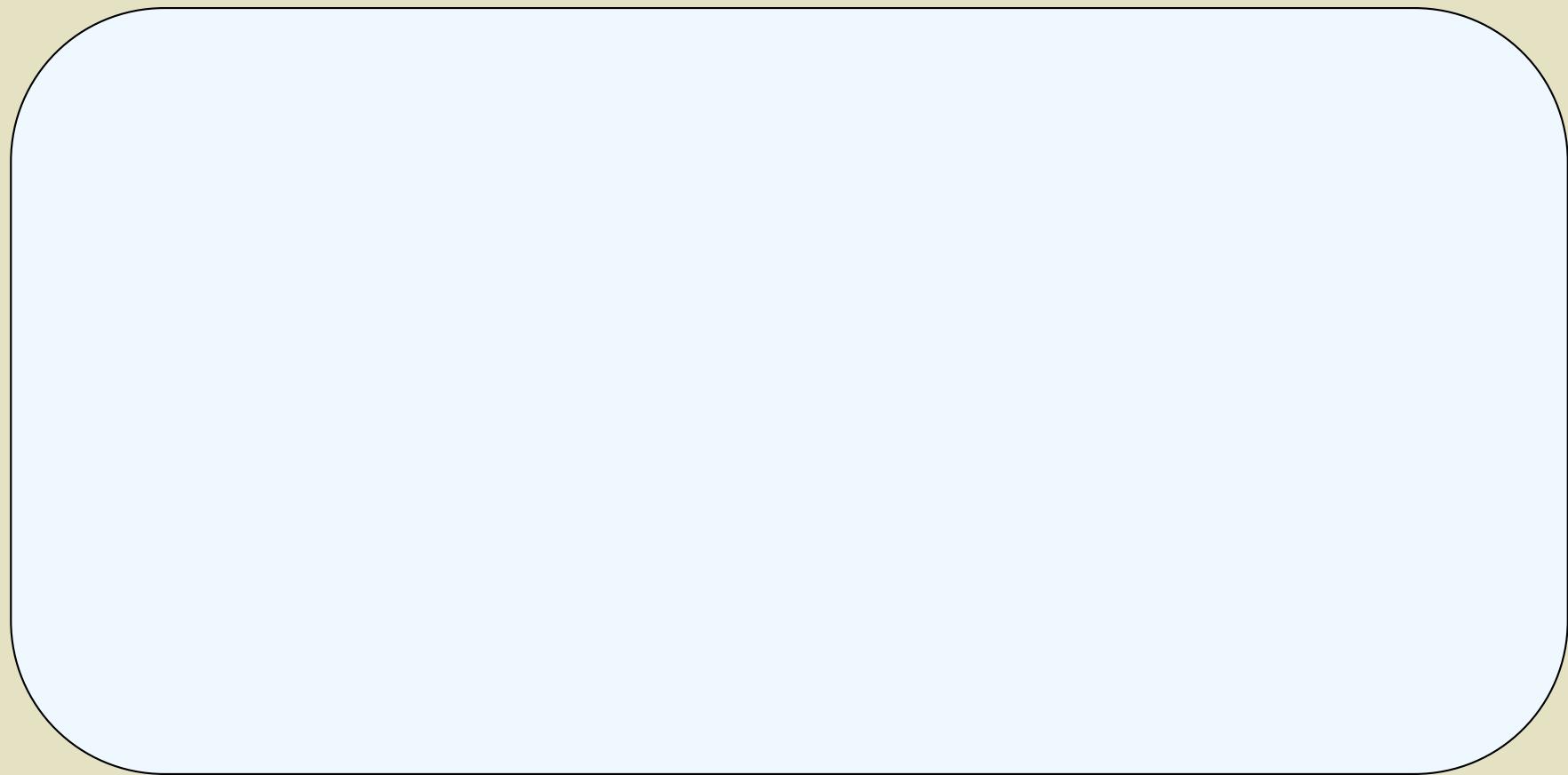
optional typing

concrete typing

...

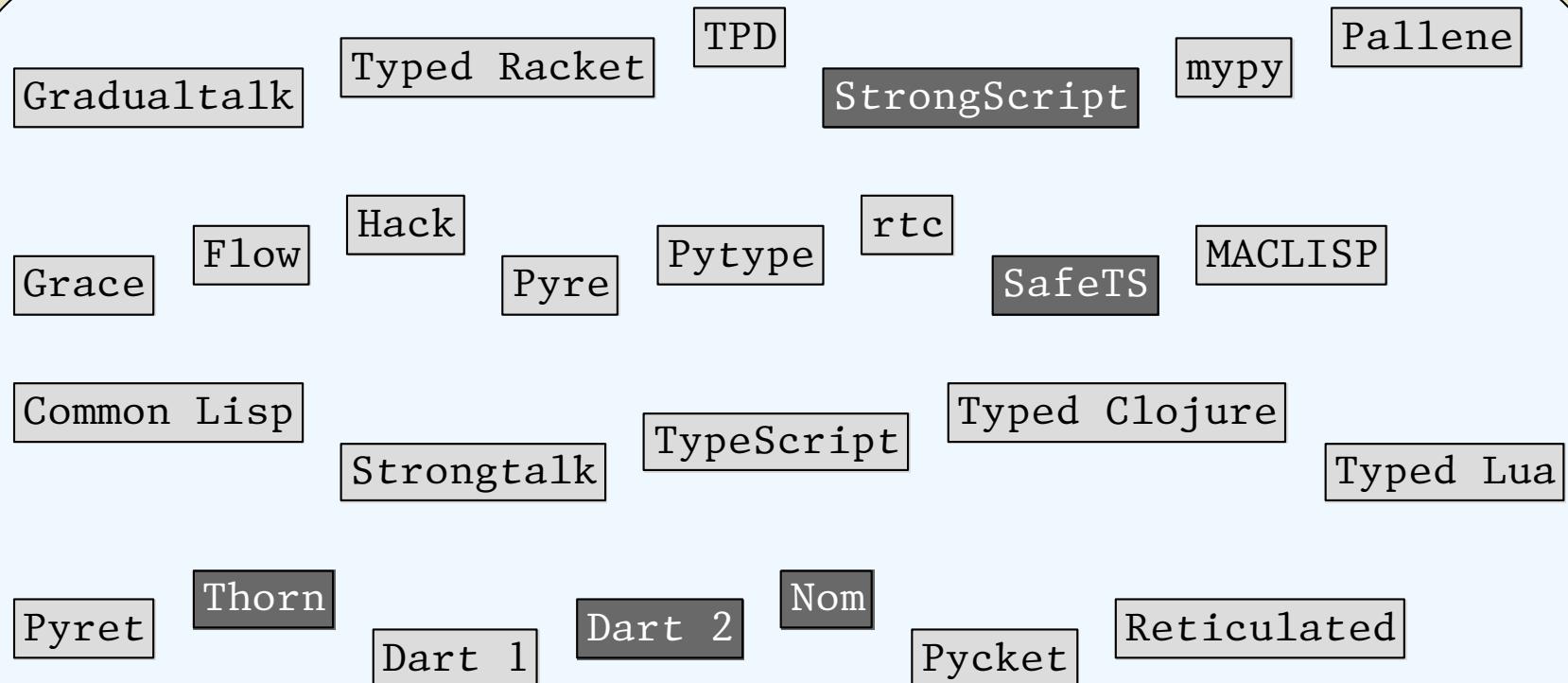
(the research landscape)

Mixed-Typed Languages



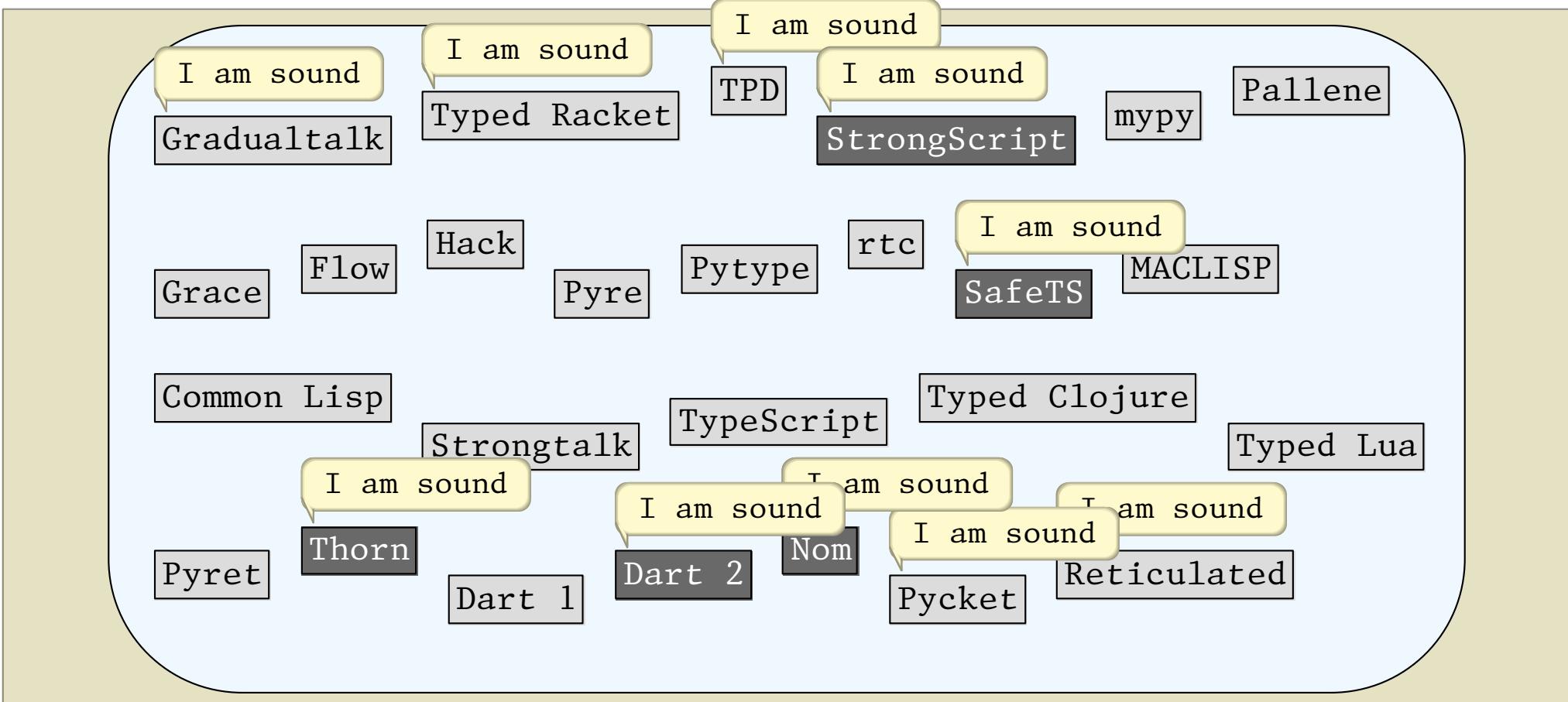
(the systems landscape)

Mixed-Typed Languages



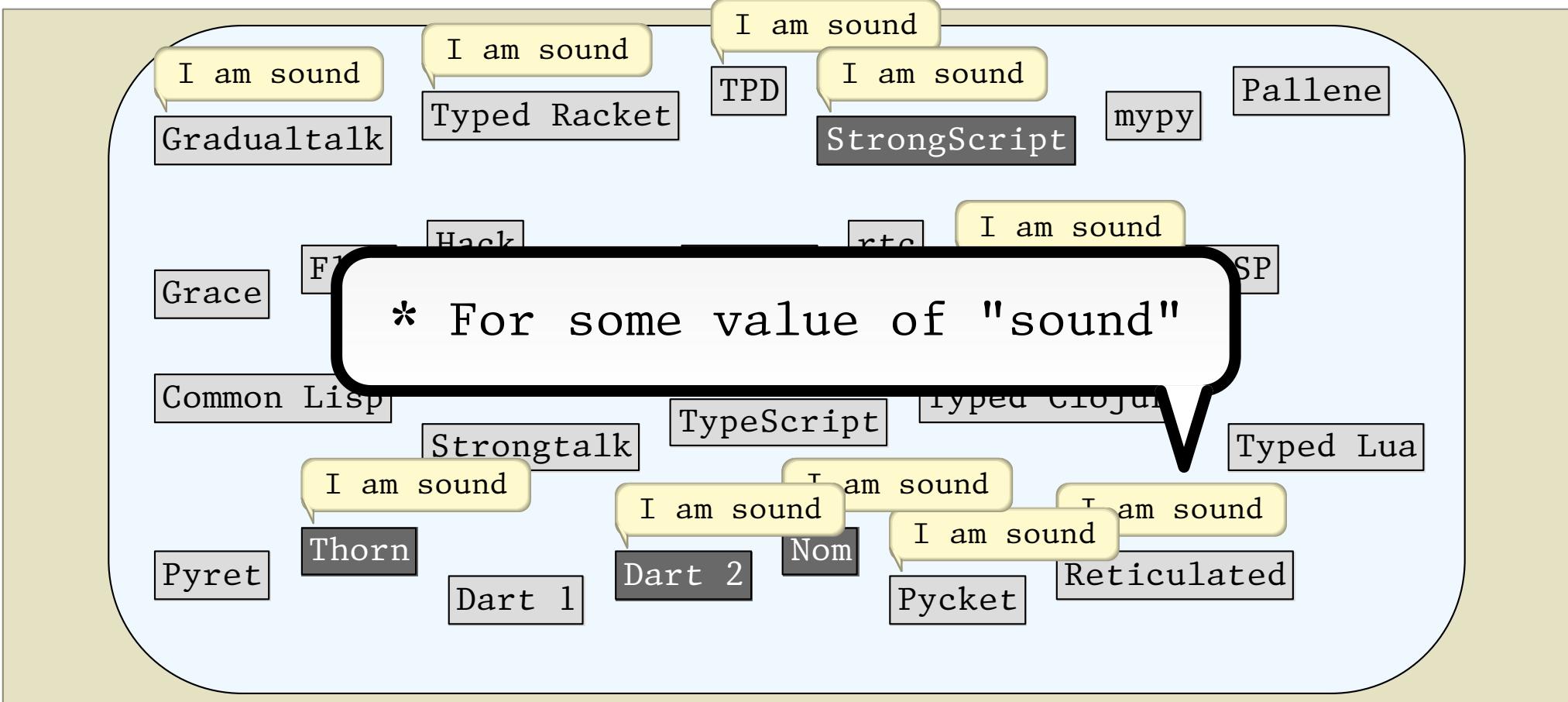
(the systems landscape)

Mixed-Typed Languages



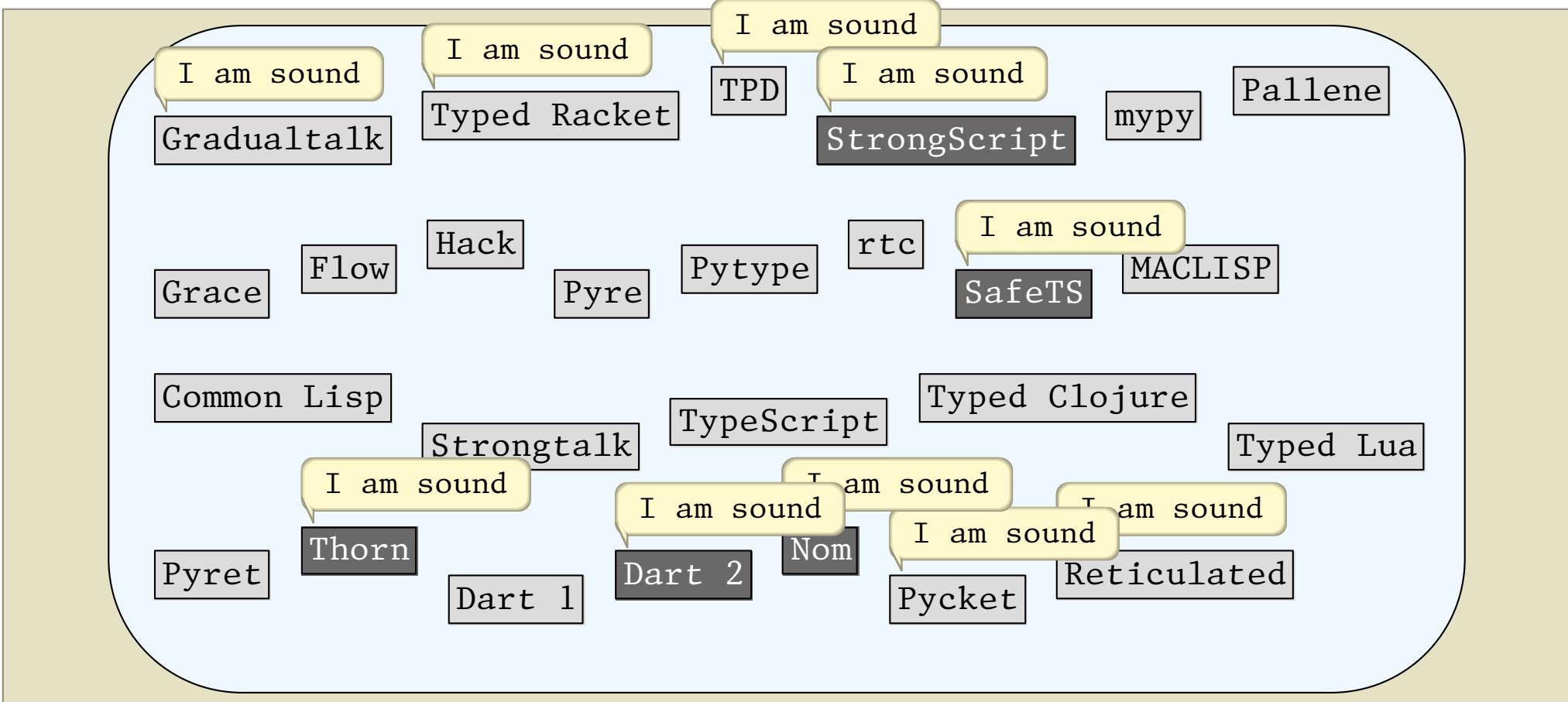
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Mixed-Typed Languages



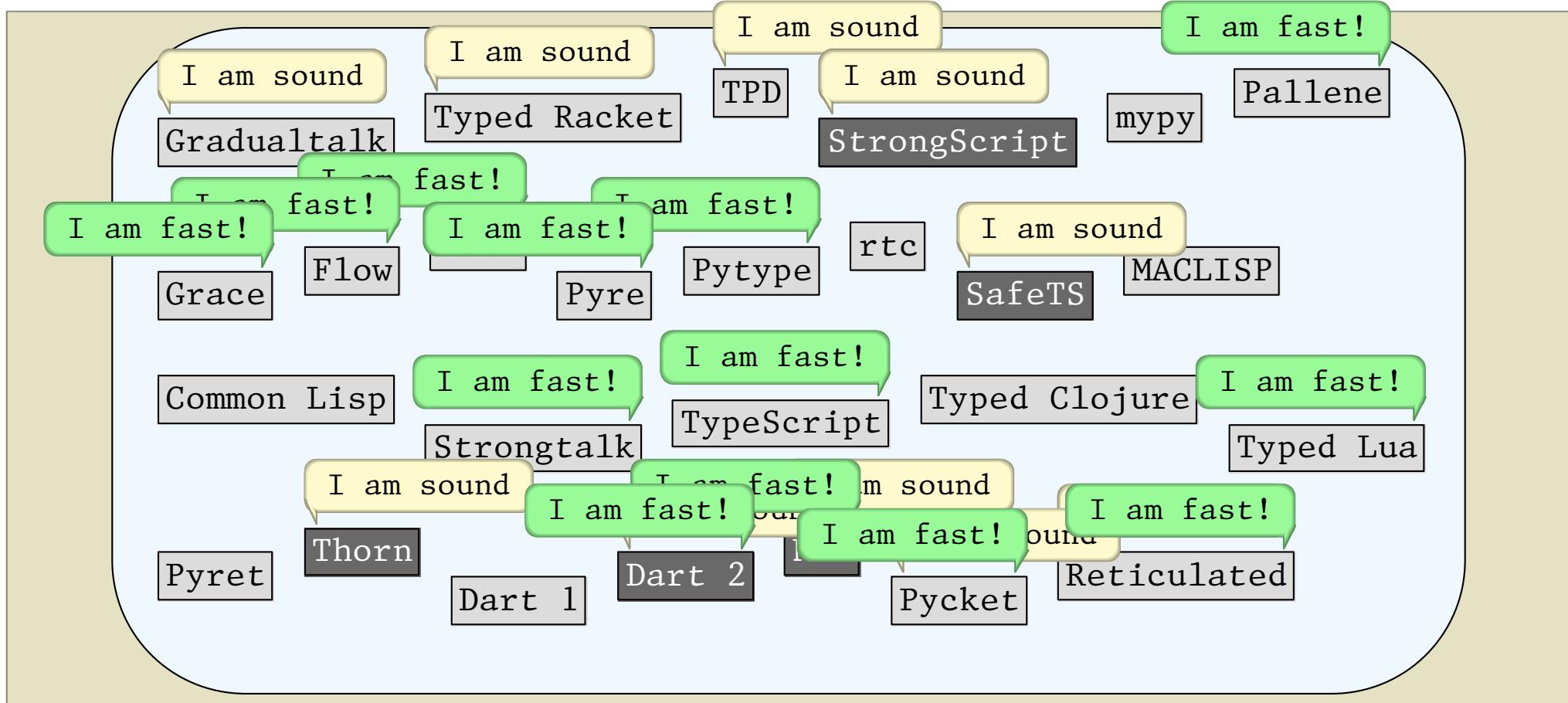
(the systems landscape)

Mixed-Typed Languages



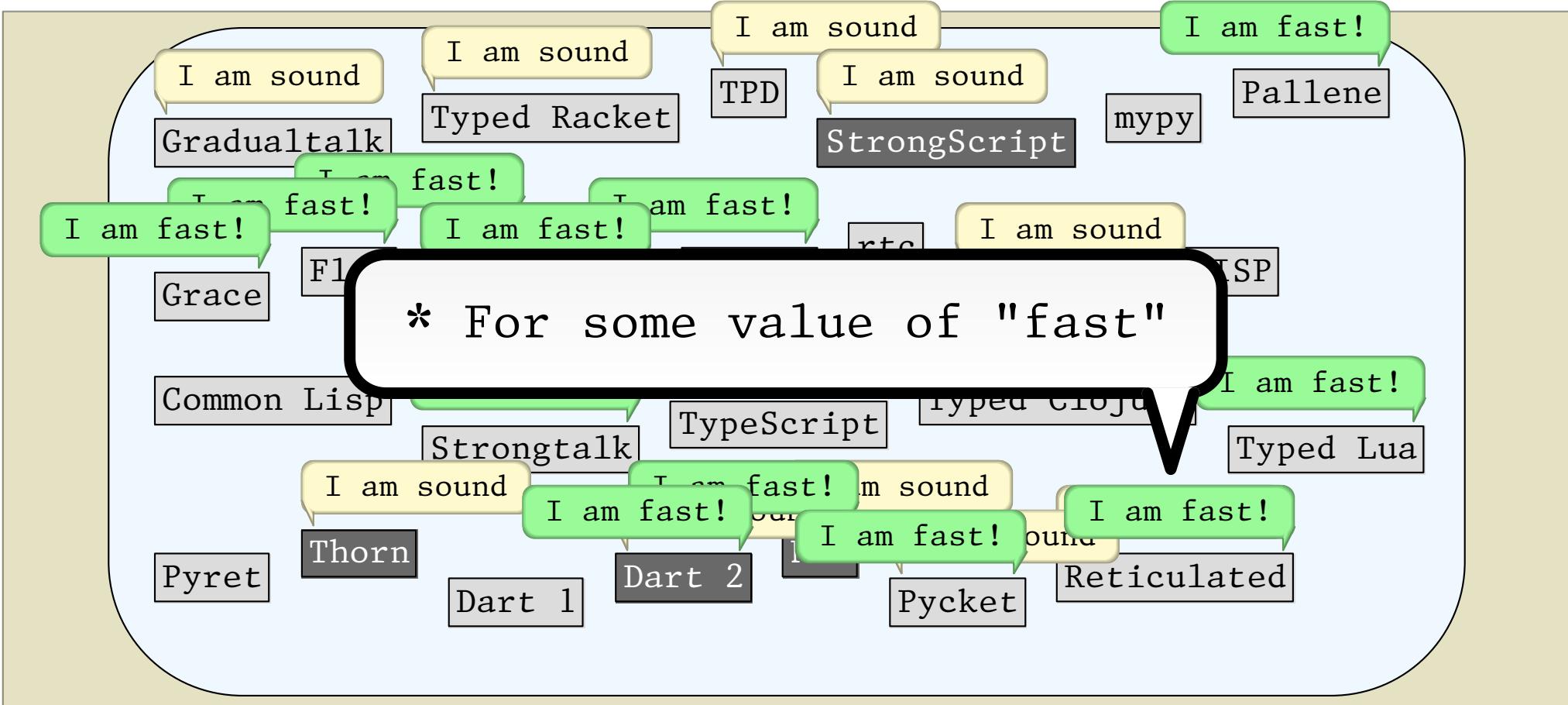
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Mixed-Typed Languages



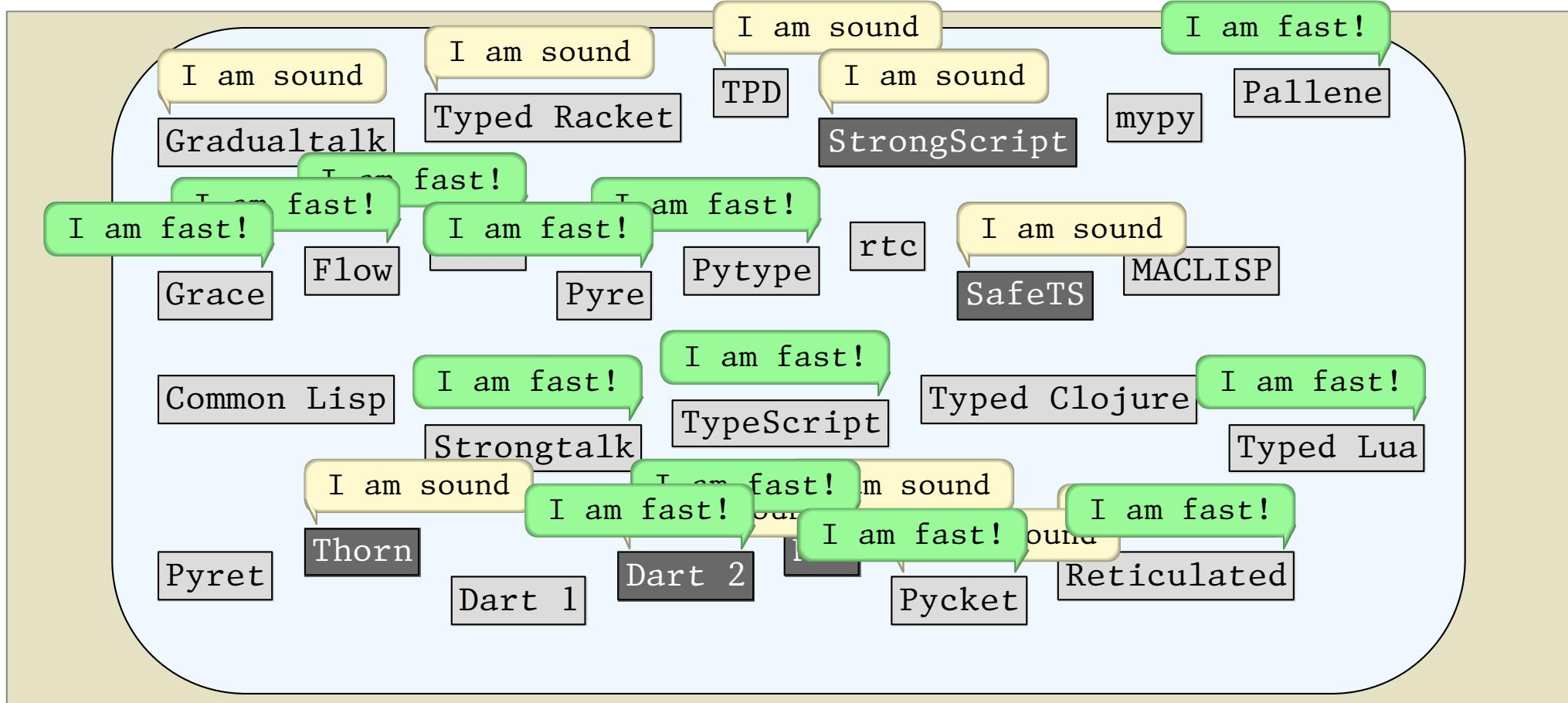
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Mixed-Typed Languages



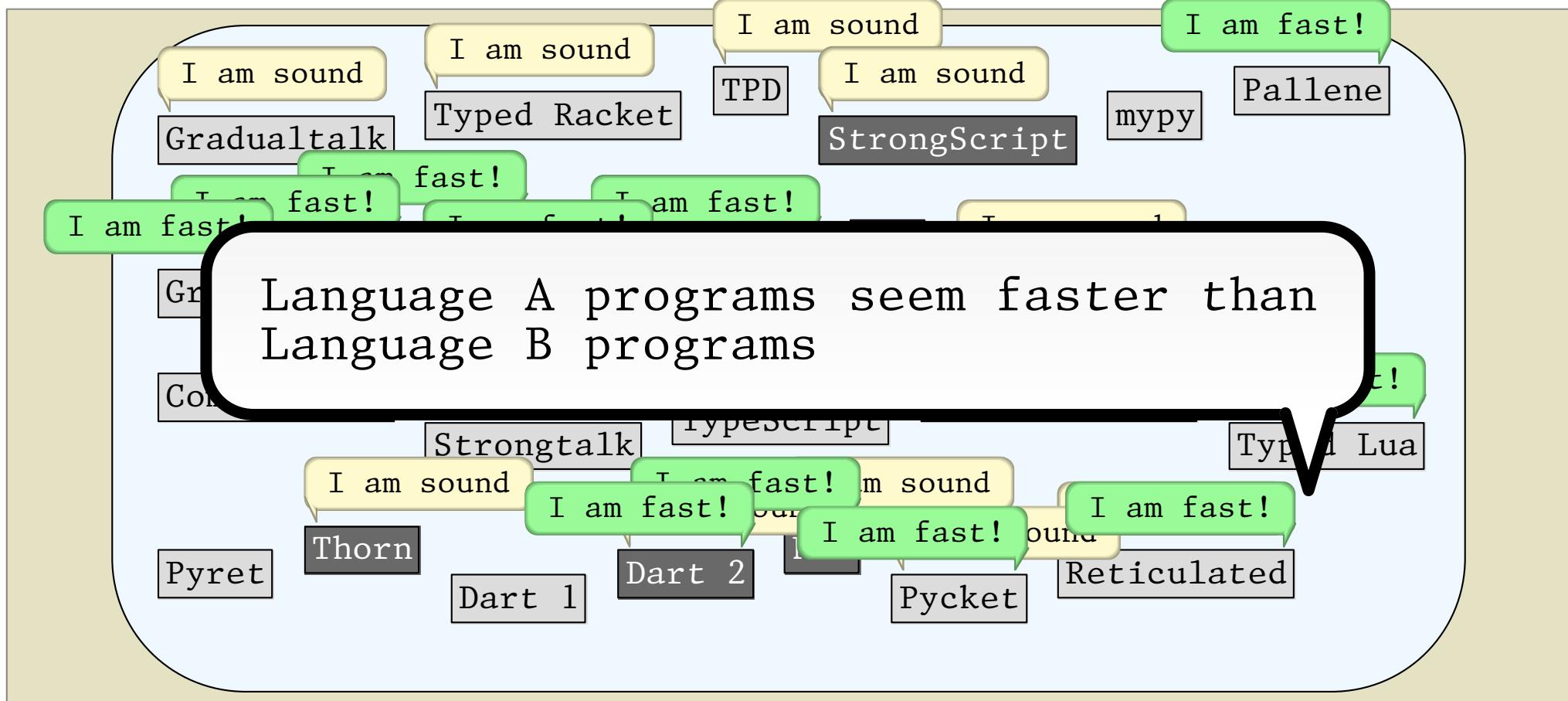
(the systems landscape)

Mixed-Typed Languages



(the systems landscape)

Mixed-Typed Languages

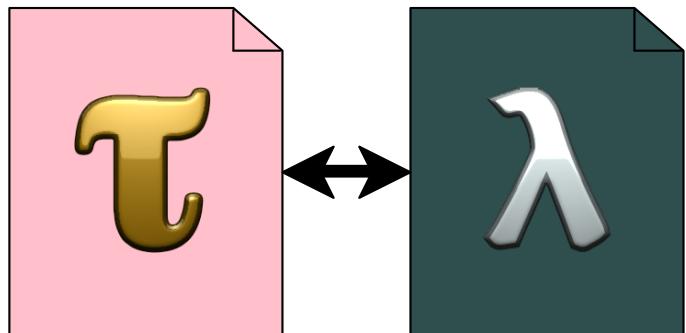


(the systems landscape)

In this paper:

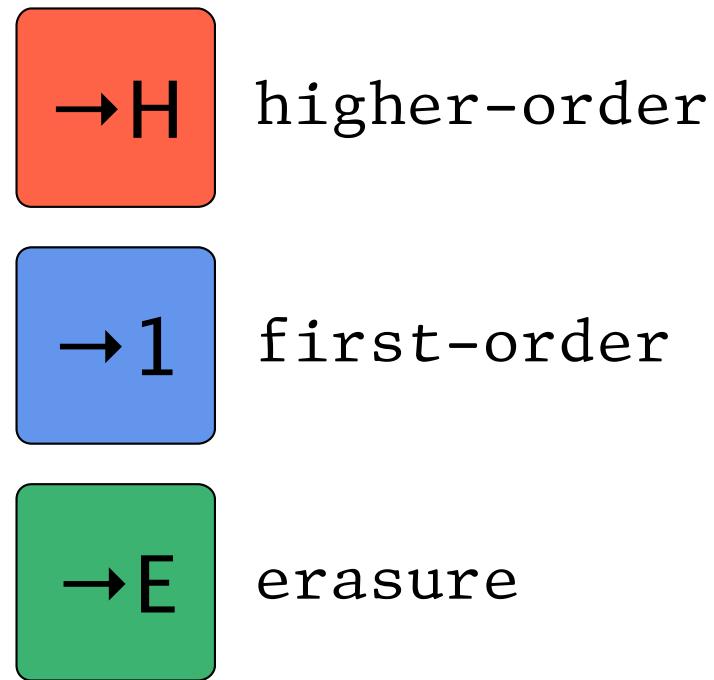
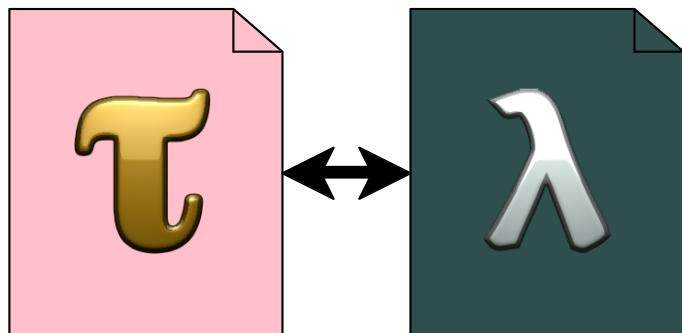
Let's put the **theory** and **practice** on
firm scientific ground.

In this paper:



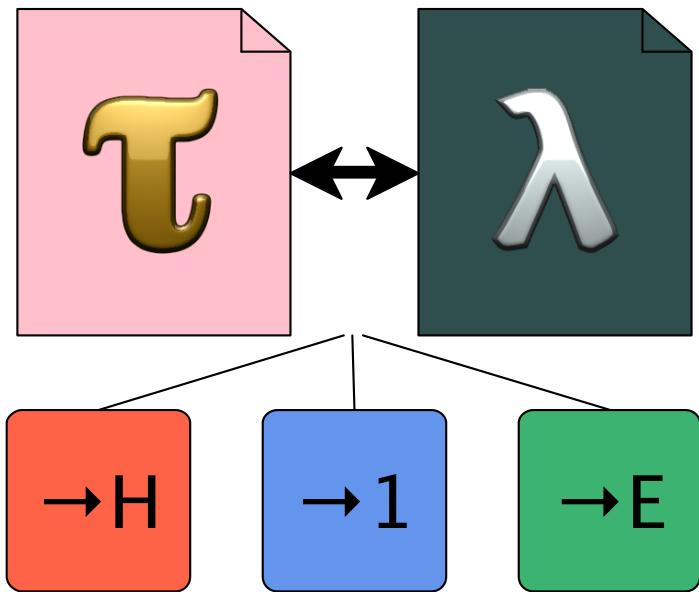
One mixed-typed language . . .

In this paper:

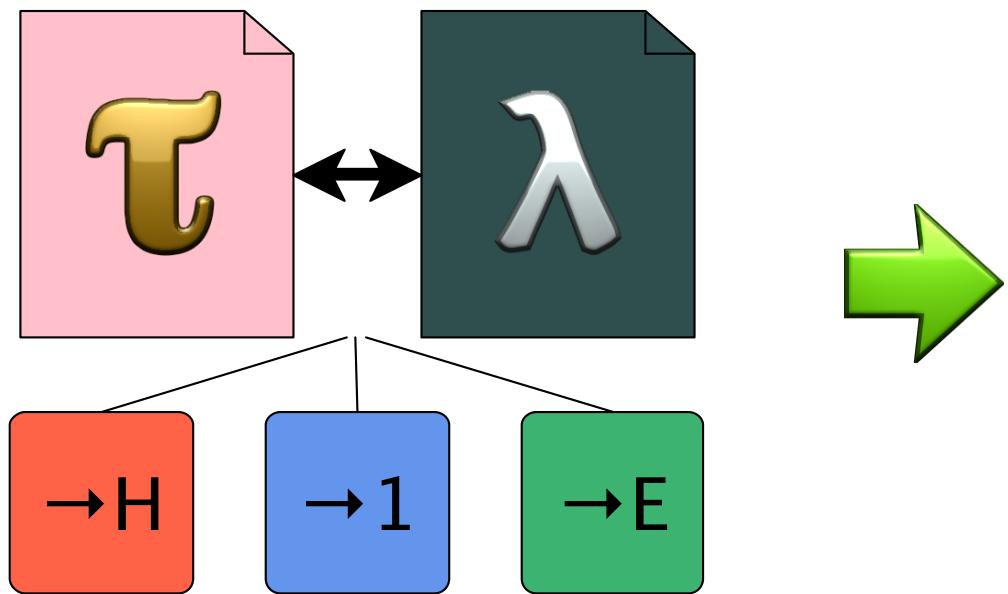


One mixed-typed language . . . three semantics

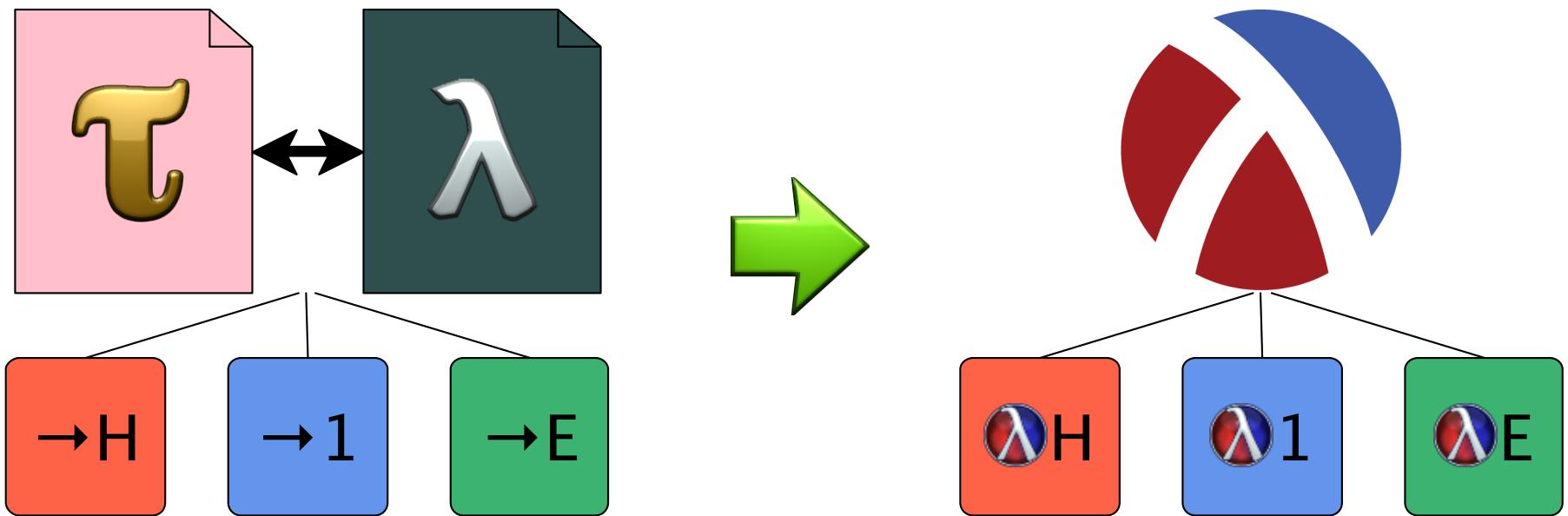
Apples-to-Apples Theory



supports direct comparisons
of the meta-theory



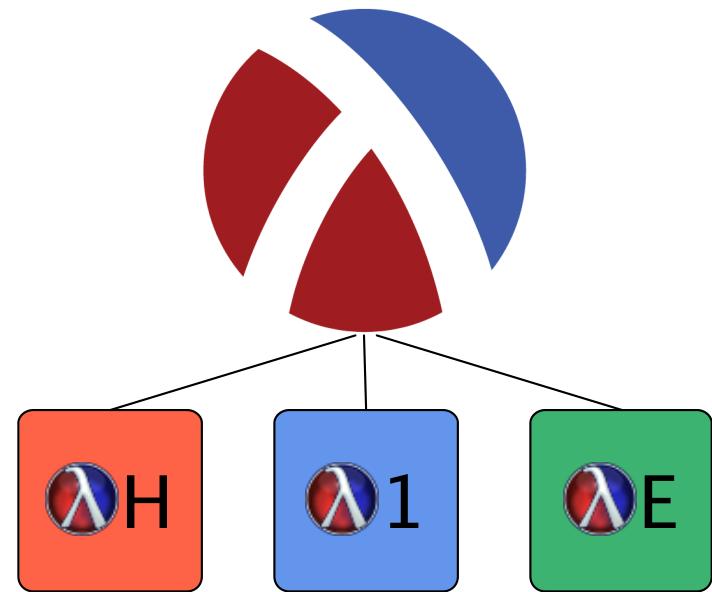
model => implementation



model => implementation

Apples-to-Apples Performance

able to systematically
compare running times



Model

$$\tau = \text{Nat} \mid \text{Int} \mid \tau \times \tau \mid \tau \Rightarrow \tau$$
$$\text{Nat} <: \text{Int}$$

$$\tau = \text{Nat} \mid \text{Int} \mid \tau \times \tau \mid \underline{\tau \Rightarrow \tau}$$

$\text{Nat} <: \text{Int}$

coinductive type

$$\tau = \text{Nat} \mid \text{Int} \mid \underline{\tau \times \tau} \mid \tau \Rightarrow \tau$$

$\text{Nat} <: \text{Int}$

inductive type

$$\tau = \text{Nat} \mid \underline{\text{Int}} \mid \tau \times \tau \mid \tau \Rightarrow \tau$$

$\text{Nat} <: \text{Int}$

base type

$$\tau = \underline{\text{Nat}} \mid \text{Int} \mid \tau \times \tau \mid \tau \Rightarrow \tau$$
$$\text{Nat} <: \text{Int}$$

base type

$$\tau = \text{Nat} \mid \text{Int} \mid \tau \times \tau \mid \tau \Rightarrow \tau$$

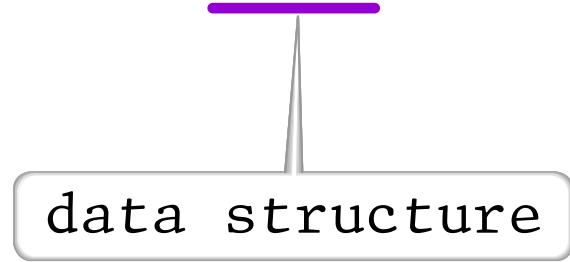
Nat <: Int

subtype relation

$$\tau = \text{Nat} \mid \text{Int} \mid \tau \times \tau \mid \tau \Rightarrow \tau$$
$$\text{Nat} <: \text{Int}$$
$$v = n \mid i \mid \langle v, v \rangle \mid \lambda(x)e$$
$$n \subset i$$

$$\tau = \text{Nat} \mid \text{Int} \mid \tau \times \tau \mid \tau \Rightarrow \tau$$
$$\text{Nat} <: \text{Int}$$
$$v = n \mid i \mid \langle v, v \rangle \mid \underline{\lambda(x)e}$$
$$n \subset i$$


higher-order value

$$\tau = \text{Nat} \mid \text{Int} \mid \tau \times \tau \mid \tau \Rightarrow \tau$$
$$\text{Nat} <: \text{Int}$$
$$v = n \mid i \mid \underline{\langle v, v \rangle} \mid \lambda(x)e$$
$$n \subset i$$


data structure

$$\tau = \text{Nat} \mid \text{Int} \mid \tau \times \tau \mid \tau \Rightarrow \tau$$
$$\text{Nat} <: \text{Int}$$
$$v = n \mid \underline{i} \mid \langle v, v \rangle \mid \lambda(x)e$$
$$n \subset i$$

base value

$$\tau = \text{Nat} \mid \text{Int} \mid \tau \times \tau \mid \tau \Rightarrow \tau$$
$$\text{Nat} <: \text{Int}$$
$$v = n \mid i \mid \langle v, v \rangle \mid \lambda(x)e$$
$$n \in \mathbb{N}$$

base value

$$\tau = \text{Nat} \mid \text{Int} \mid \tau \times \tau \mid \tau \Rightarrow \tau$$
$$\text{Nat} <: \text{Int}$$
$$v = n \mid i \mid \langle v, v \rangle \mid \lambda(x)e$$
$$\underline{n \subset i}$$

subset relation

$$\tau = \text{Nat} \mid \text{Int} \mid \tau \times \tau \mid \tau \Rightarrow \tau$$
$$\text{Nat} <: \text{Int}$$
$$v = n \mid i \mid \langle v, v \rangle \mid \lambda(x)e$$
$$n \subset i$$
$$e = \dots \mid \text{dyn } \tau e \mid \text{stat } \tau e$$

$$\tau = \text{Nat} \mid \text{Int} \mid \tau \times \tau \mid \tau \Rightarrow \tau$$
$$\text{Nat} <: \text{Int}$$
$$v = n \mid i \mid \langle v, v \rangle \mid \lambda(x)e$$
$$n \subset i$$
$$e = \dots \mid \underline{\text{dyn } \tau e} \mid \underline{\text{stat } \tau e}$$

boundary terms

$$\tau = \text{Nat} \mid \text{Int} \mid \tau \times \tau \mid \tau \Rightarrow \tau$$
$$\text{Nat} <: \text{Int}$$
$$v = n \mid i \mid \langle v, v \rangle \mid \lambda(x)e$$
$$n \subset i$$
$$e = \dots \mid \text{dyn } \tau e \mid \text{stat } \tau e$$

$$\tau = \text{Nat} \mid \text{Int} \mid \tau \times \tau \mid \tau \Rightarrow \tau$$
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$$v = n \mid i \mid \langle v, v \rangle \mid \lambda(x)e$$
$$n \subset i$$
$$e = \dots \mid \text{dyn } \tau e \mid \text{stat } \tau e$$
$$\boxed{\vdash e : \tau}$$
$$\vdash e$$
$$\boxed{\vdash e}$$
$$\vdash e : \tau$$
$$\vdash \text{dyn } \tau e : \tau$$
$$\vdash \text{stat } \tau e$$

```
fib   : Nat ̗ Nat
Γ = norm : Nat × Nat ̗ Nat
      map : (Nat ̗ Nat) ̗ Nat × Nat ̗ Nat × Nat

Γ ⊢ fib (dyn Nat -1) : Nat
Γ ⊢ norm (dyn Nat × Nat <-1,-2>) : Nat
Γ ⊢ map (dyn (Nat ̗ Nat) (λ(x)-x)) y : Nat × Nat
```

```
fib   : Nat ̗ Nat  
Γ = norm : Nat × Nat ̗ Nat  
map   : (Nat ̗ Nat) ̗ Nat × Nat ̗ Nat × Nat
```

```
Γ ⊢ fib  (dyn Nat -1) : Nat  
Γ ⊢ norm (dyn Nat × Nat <-1,-2>) : Nat  
Γ ⊢ map  (dyn (Nat ̗ Nat) (λ(x)-x)) y : Nat × Nat
```

```
fib   : Nat  ⇒ Nat
 $\Gamma =$  norm : Nat × Nat  ⇒ Nat
 $\Gamma$ 
map   : (Nat  ⇒ Nat)  ⇒ Nat × Nat  ⇒ Nat × Nat
```

```
 $\Gamma \vdash \text{fib } (\text{dyn Nat } -1) : \text{Nat}$ 
 $\Gamma \vdash \text{norm } (\text{dyn Nat } \times \text{Nat } \langle -1, -2 \rangle) : \text{Nat}$ 
 $\Gamma \vdash \text{map } (\text{dyn } (\text{Nat } \Rightarrow \text{Nat}) \ (\lambda(x)-x)) \ y : \text{Nat } \times \text{Nat}$ 
```

$\text{fib} : \text{Nat} \Rightarrow \text{Nat}$

$\underline{\Gamma = \text{norm} : \text{Nat} \times \text{Nat} \Rightarrow \text{Nat}}$

$\text{map} : (\text{Nat} \Rightarrow \text{Nat}) \Rightarrow \text{Nat} \times \text{Nat} \Rightarrow \text{Nat} \times \text{Nat}$

$\Gamma \vdash \text{fib} (\text{dyn Nat } -1) : \text{Nat}$

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$\Gamma \vdash \text{map} (\text{dyn } (\text{Nat} \Rightarrow \text{Nat}) \ (\lambda(x)-x)) \ y : \text{Nat} \times \text{Nat}$

`fib : Nat \Rightarrow Nat`

$\Gamma = \text{norm} : \text{Nat} \times \text{Nat} \Rightarrow \text{Nat}$

`map : (Nat \Rightarrow Nat) \Rightarrow Nat \times Nat \Rightarrow Nat \times Nat`

$\Gamma \vdash \text{fib} (\text{dyn Nat } -1) : \text{Nat}$

$\Gamma \vdash \text{norm} (\text{dyn Nat} \times \text{Nat} \langle -1, -2 \rangle) : \text{Nat}$

$\Gamma \vdash \text{map} (\text{dyn} (\text{Nat} \Rightarrow \text{Nat}) (\lambda(x)-x)) y : \text{Nat} \times \text{Nat}$

`fib : Nat \Rightarrow Nat`

$\Gamma =$ `norm : Nat \times Nat \Rightarrow Nat`

`map : (Nat \Rightarrow Nat) \Rightarrow Nat \times Nat \Rightarrow Nat \times Nat`

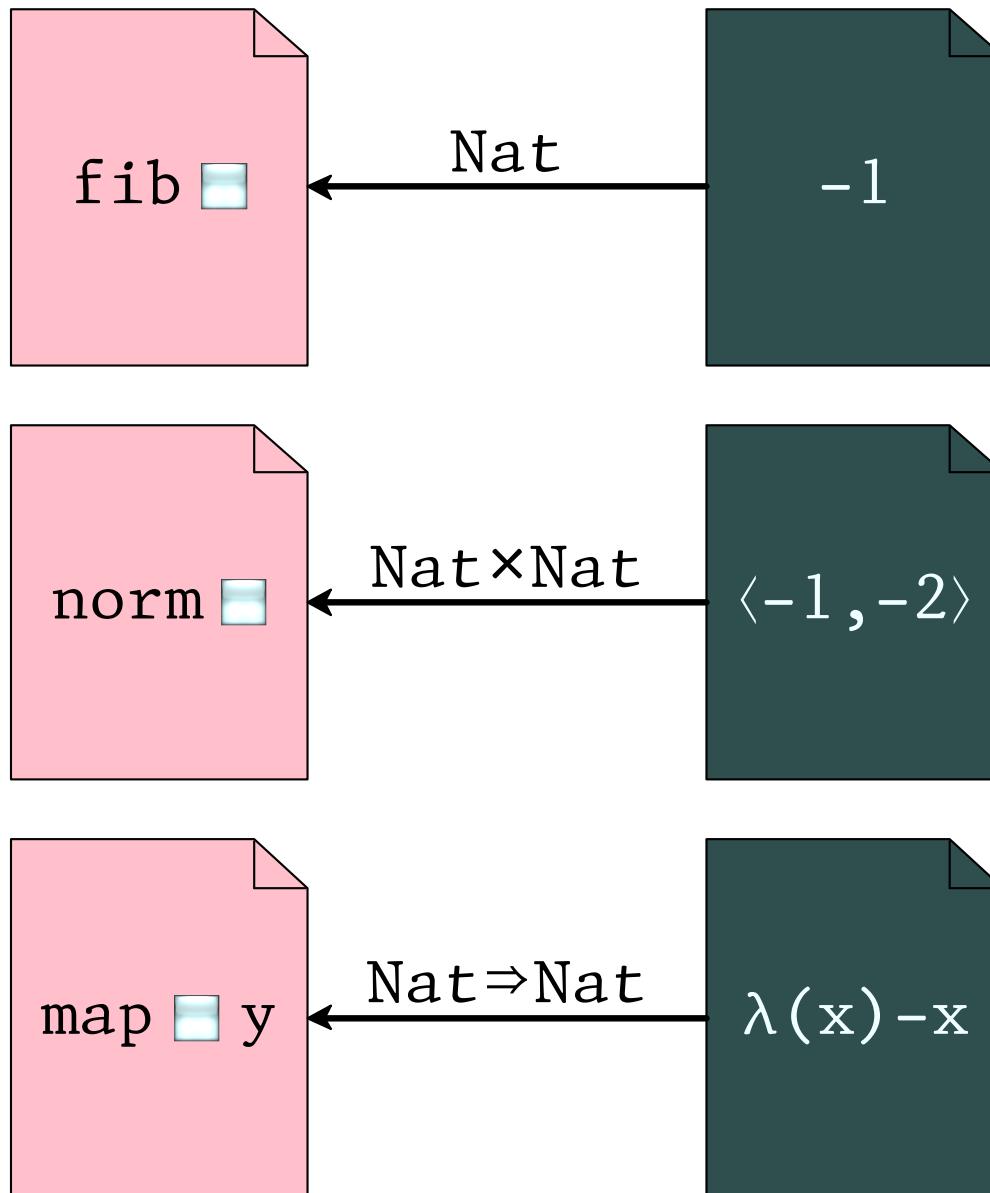
$\Gamma \vdash \text{fib } (\text{dyn Nat } -1) : \text{Nat}$

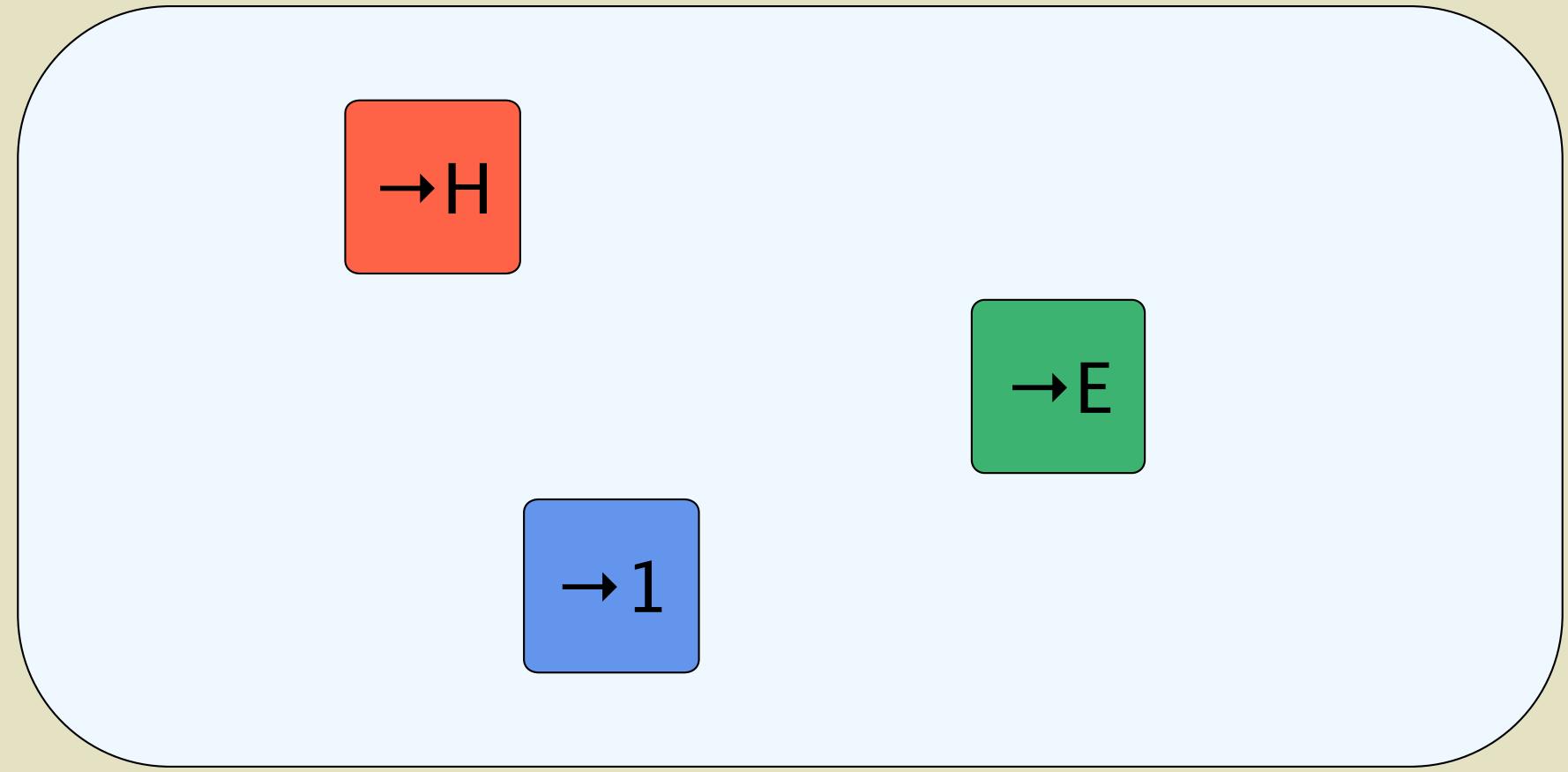
$\Gamma \vdash \text{norm } (\text{dyn Nat } \times \text{Nat } \langle -1, -2 \rangle) : \text{Nat}$

$\Gamma \vdash \text{map } (\text{dyn } (\text{Nat } \Rightarrow \text{Nat}) \ (\lambda(x)-x)) \ y : \text{Nat } \times \text{Nat}$

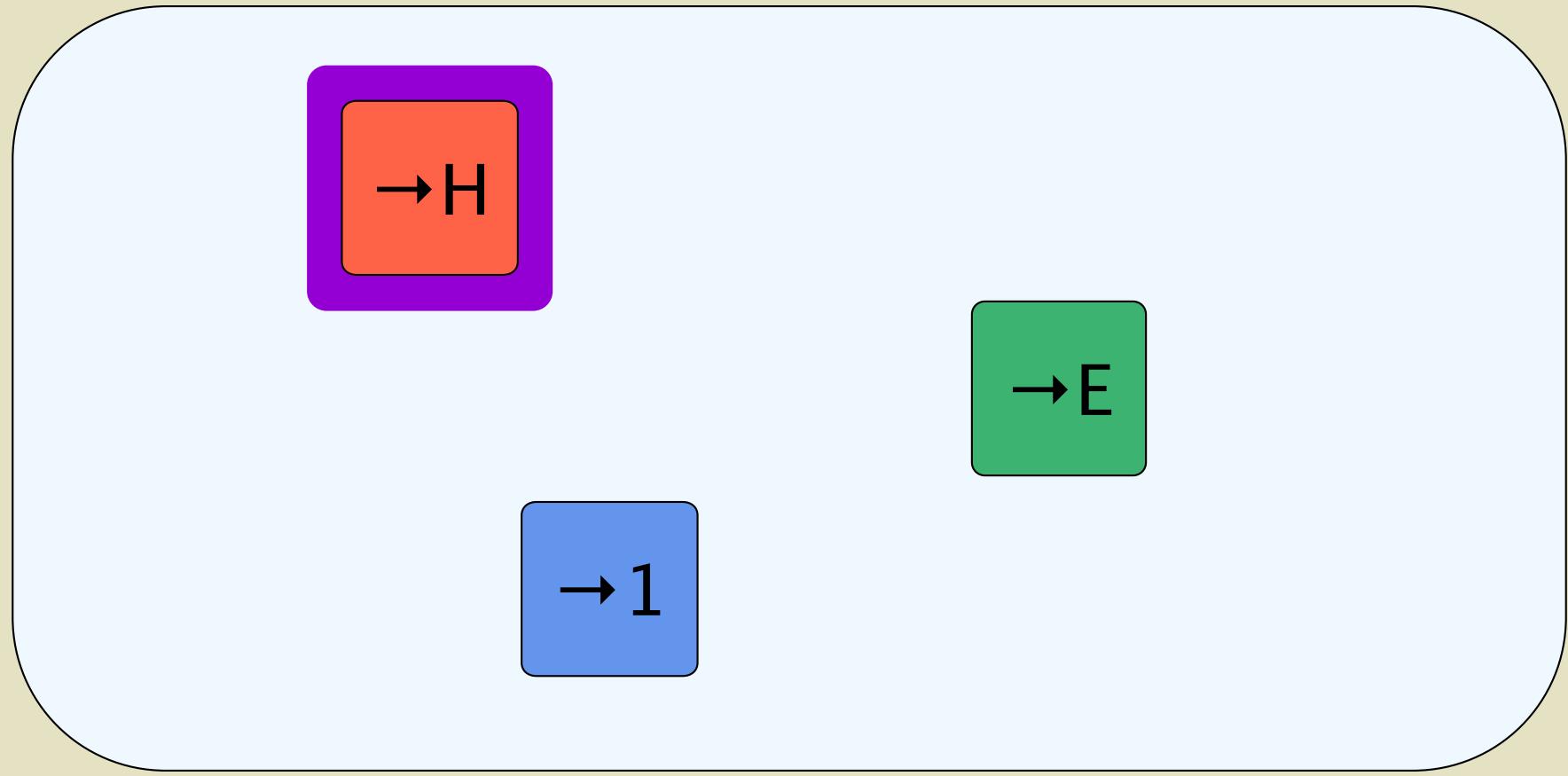
$\text{fib} : \text{Nat} \Rightarrow \text{Nat}$
 $\Gamma = \text{norm} : \text{Nat} \times \text{Nat} \Rightarrow \text{Nat}$
 $\text{map} : (\text{Nat} \Rightarrow \text{Nat}) \Rightarrow \text{Nat} \times \text{Nat} \Rightarrow \text{Nat} \times \text{Nat}$

$\Gamma \vdash \text{fib} \ \underline{(\text{dyn Nat } -1)} : \text{Nat}$
 $\Gamma \vdash \text{norm} \ \underline{(\text{dyn Nat } \times \text{Nat } \langle -1, -2 \rangle)} : \text{Nat}$
 $\Gamma \vdash \text{map} \ \underline{(\text{dyn } (\text{Nat} \Rightarrow \text{Nat}) \ (\lambda(x)-x)) \ y} : \text{Nat} \times \text{Nat}$



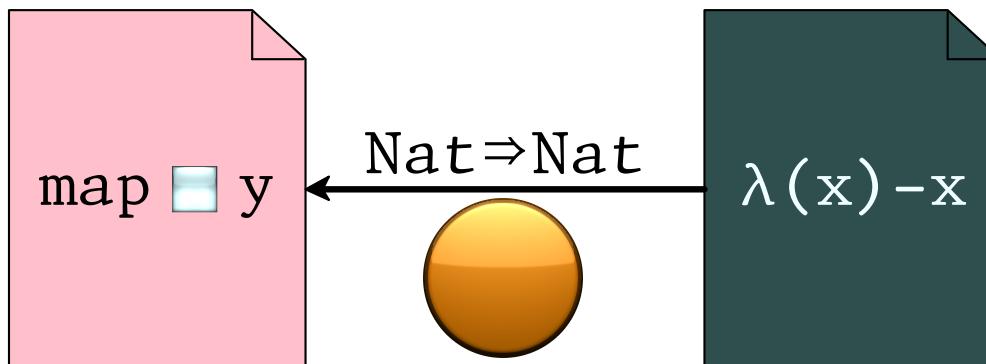
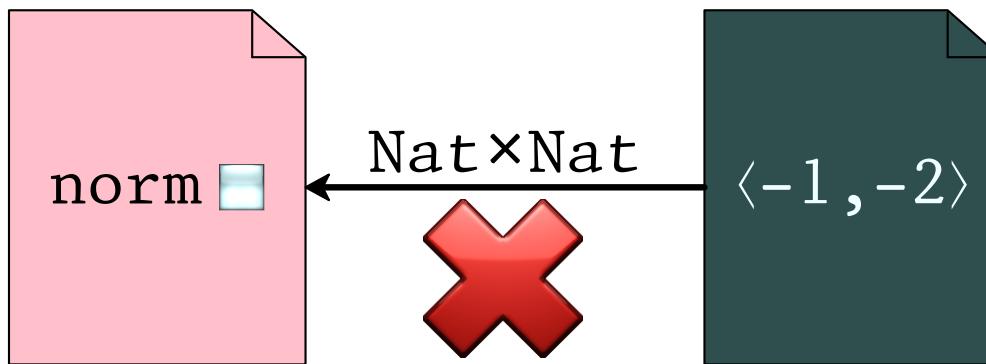
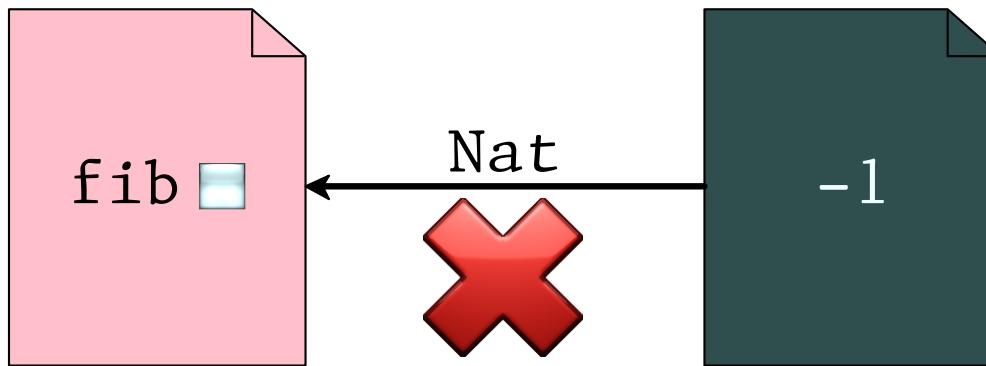
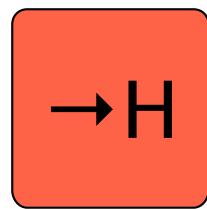


(the systems landscape)

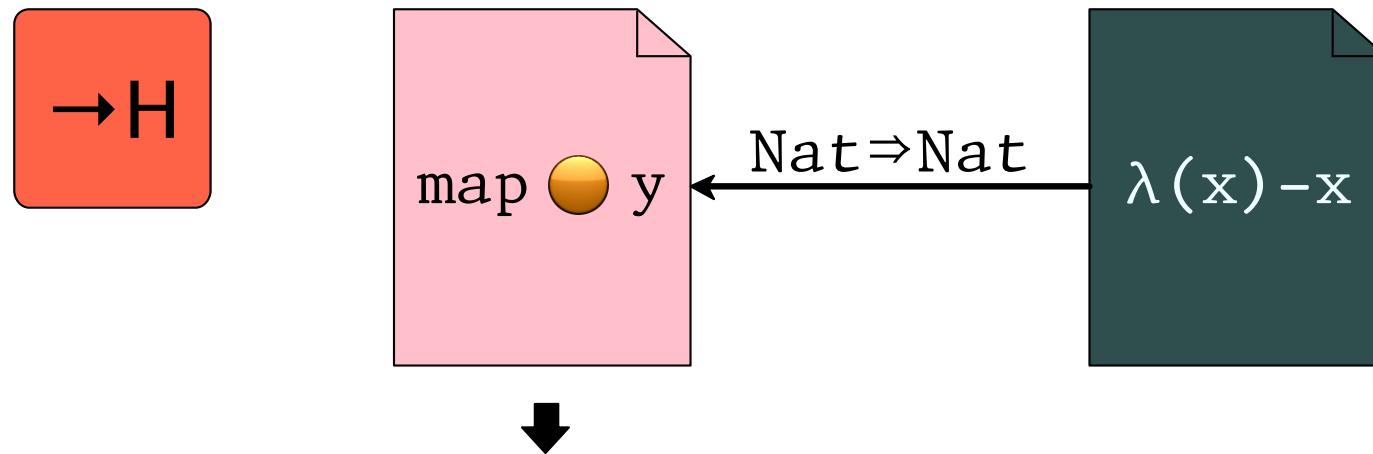


(the systems landscape)

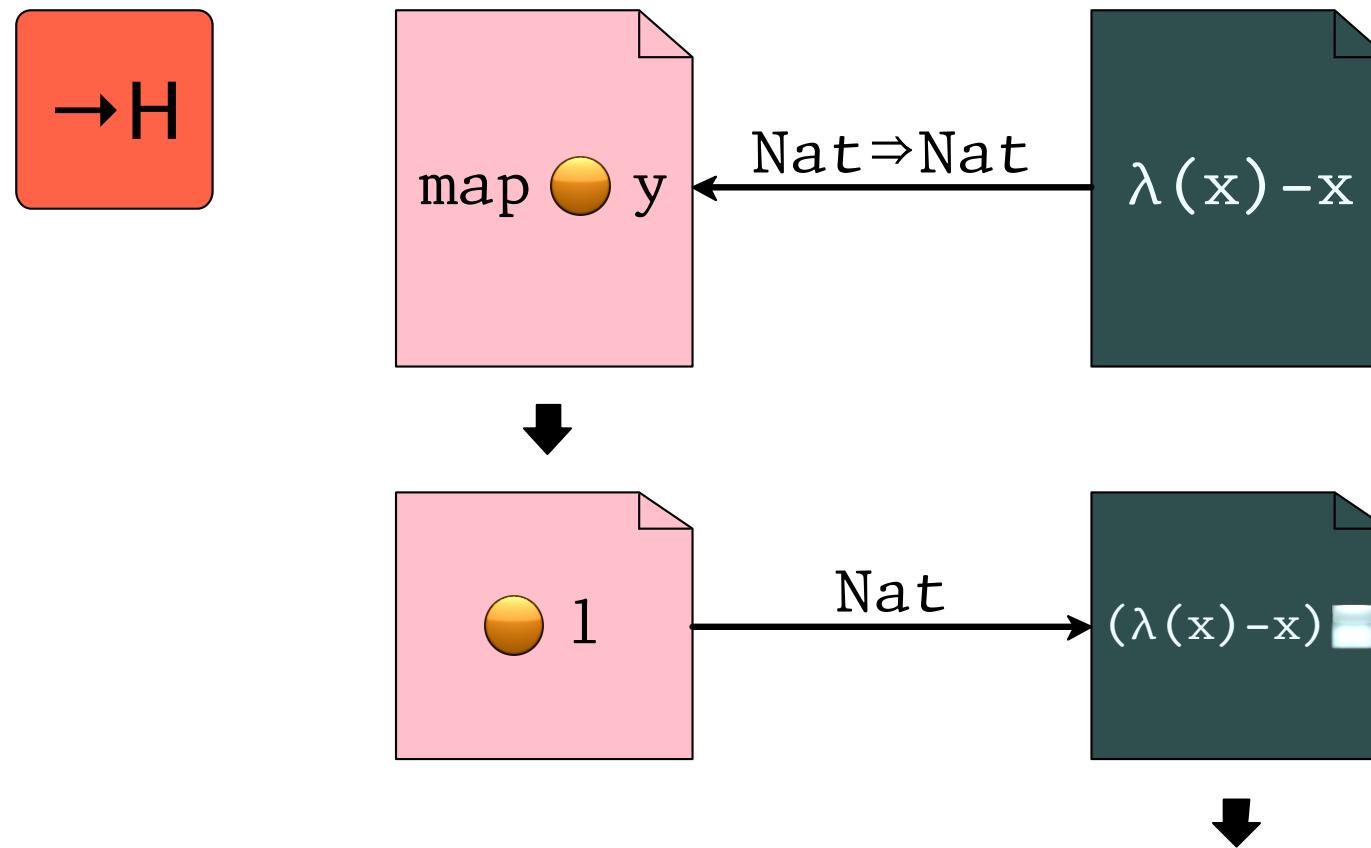
higher-order (enforce full types)



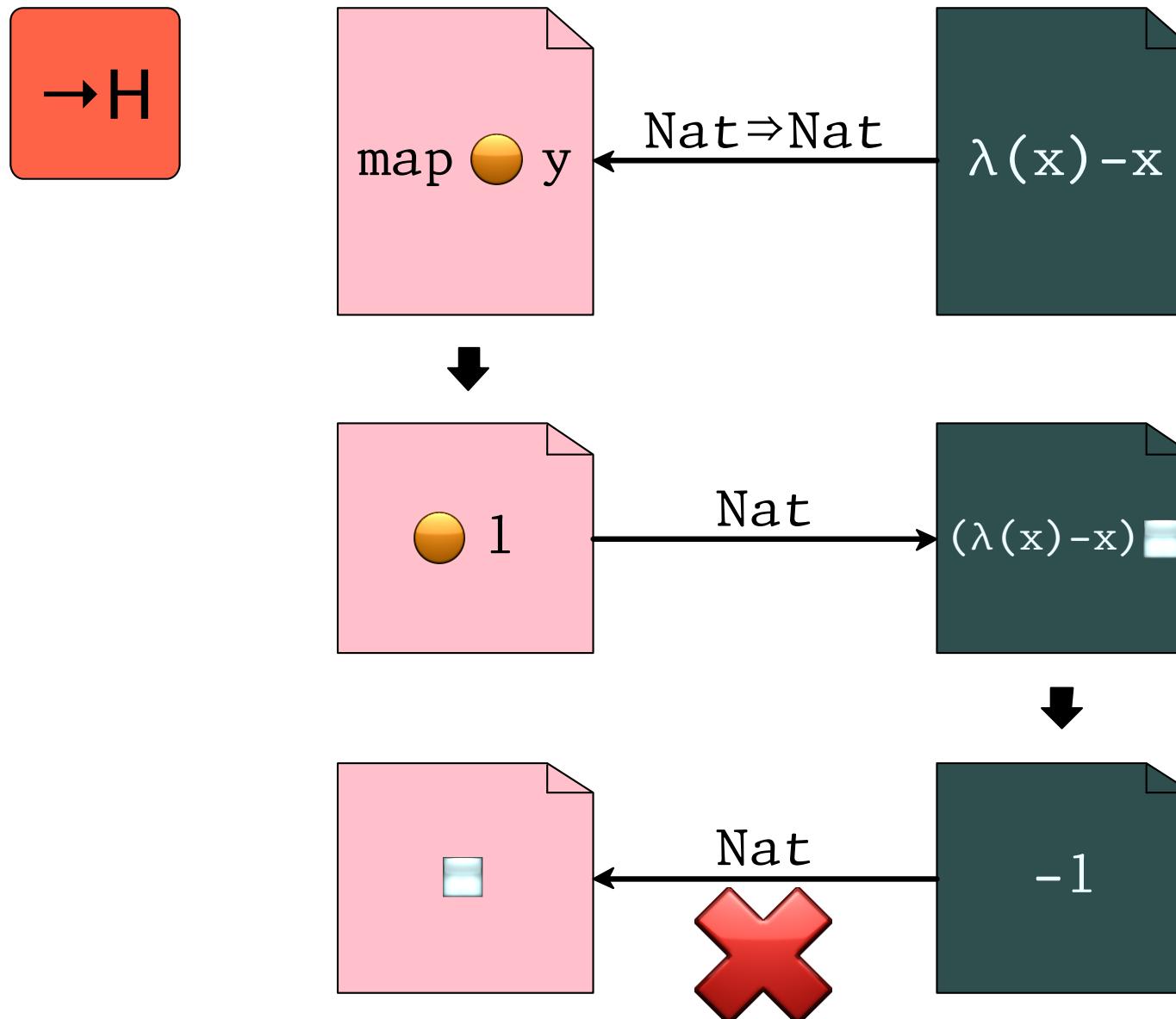
higher-order (enforce full types)

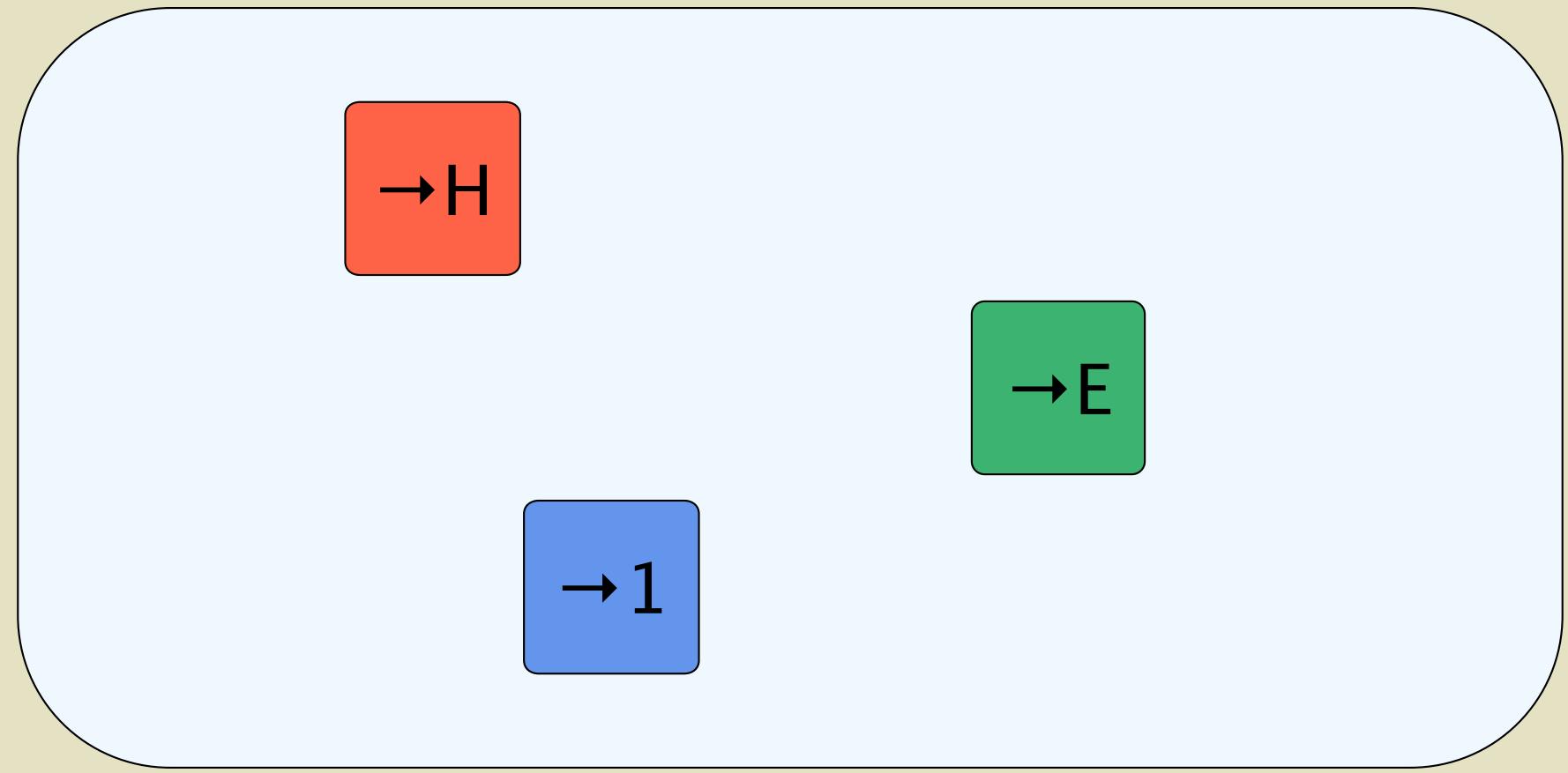


higher-order (enforce full types)

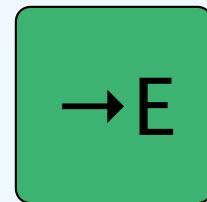
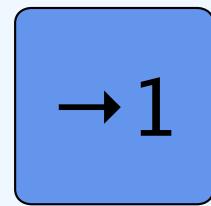
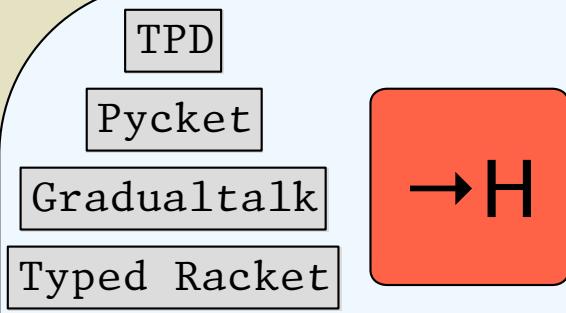


higher-order (enforce full types)

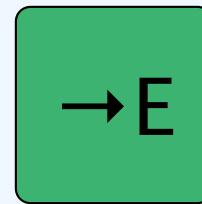
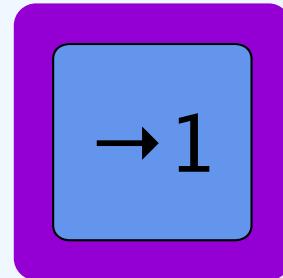
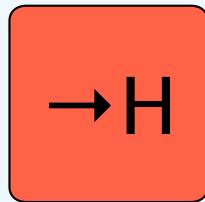
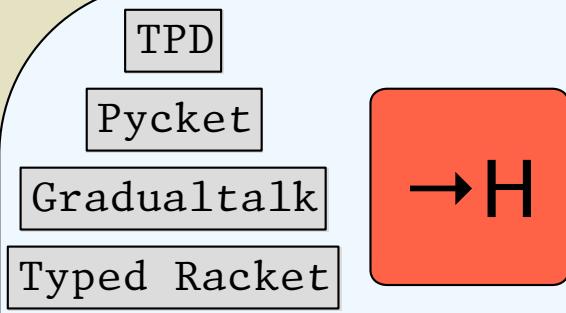




(the systems landscape)

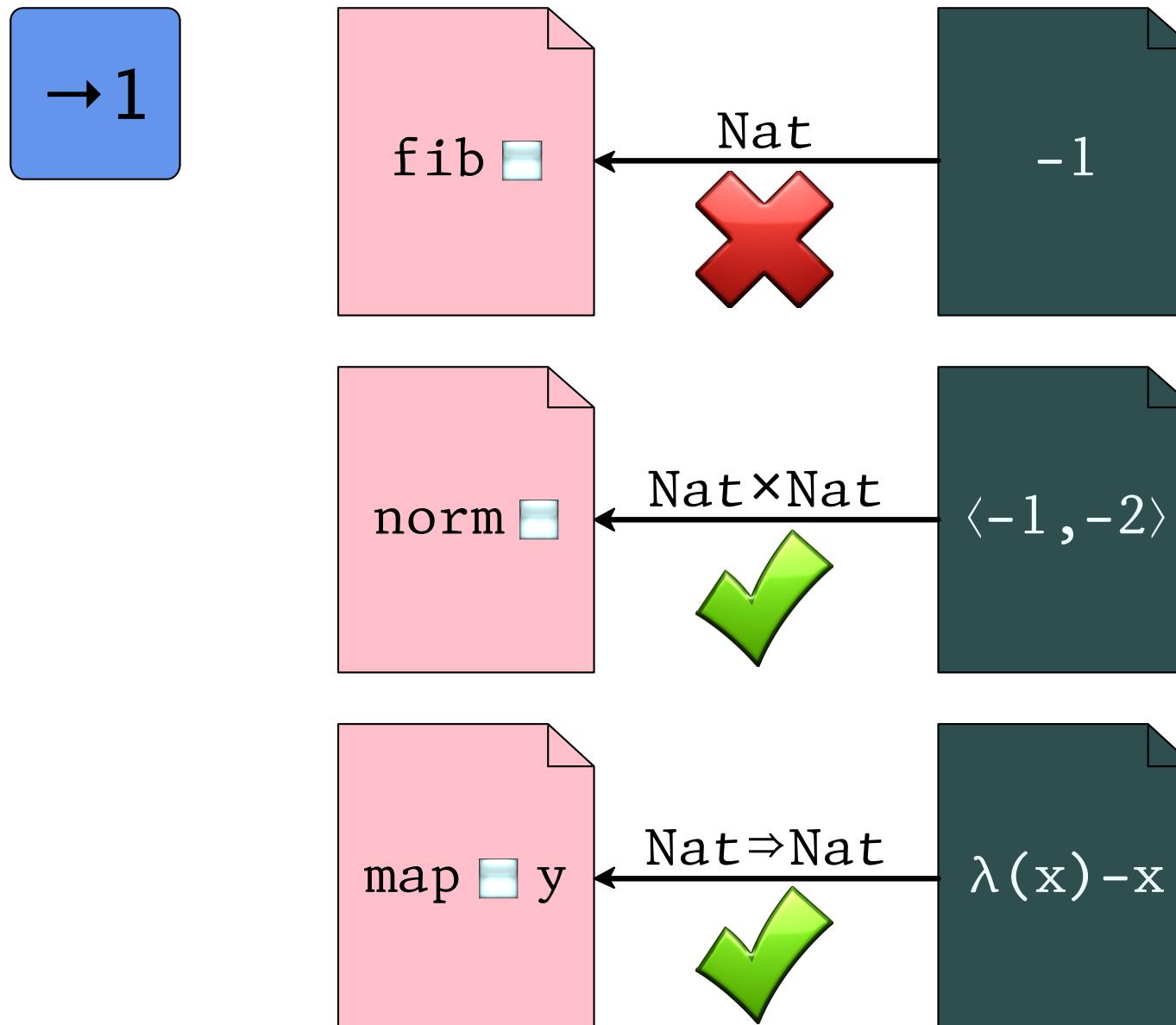


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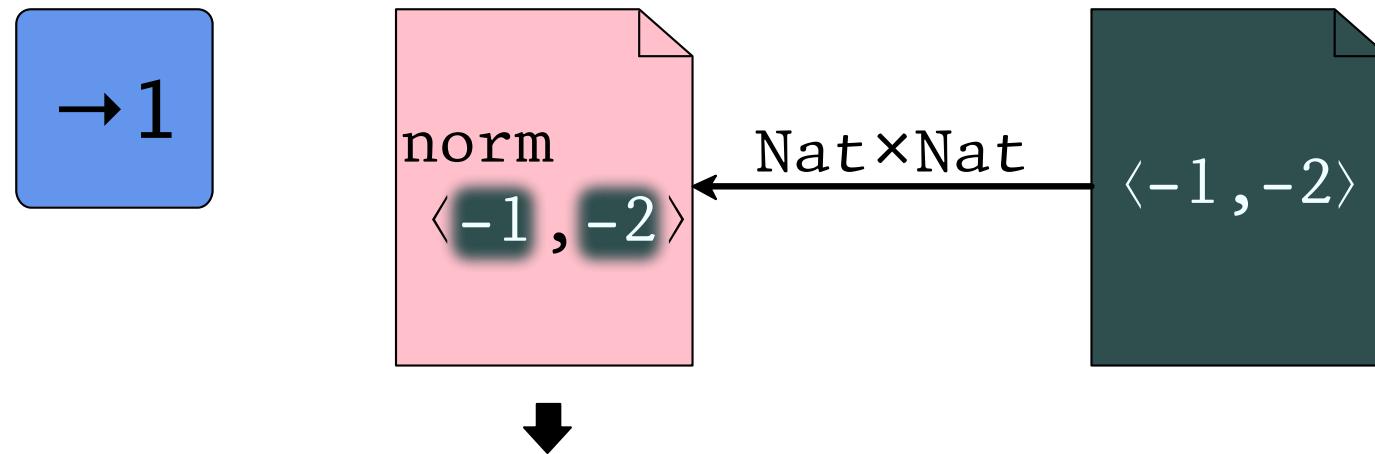


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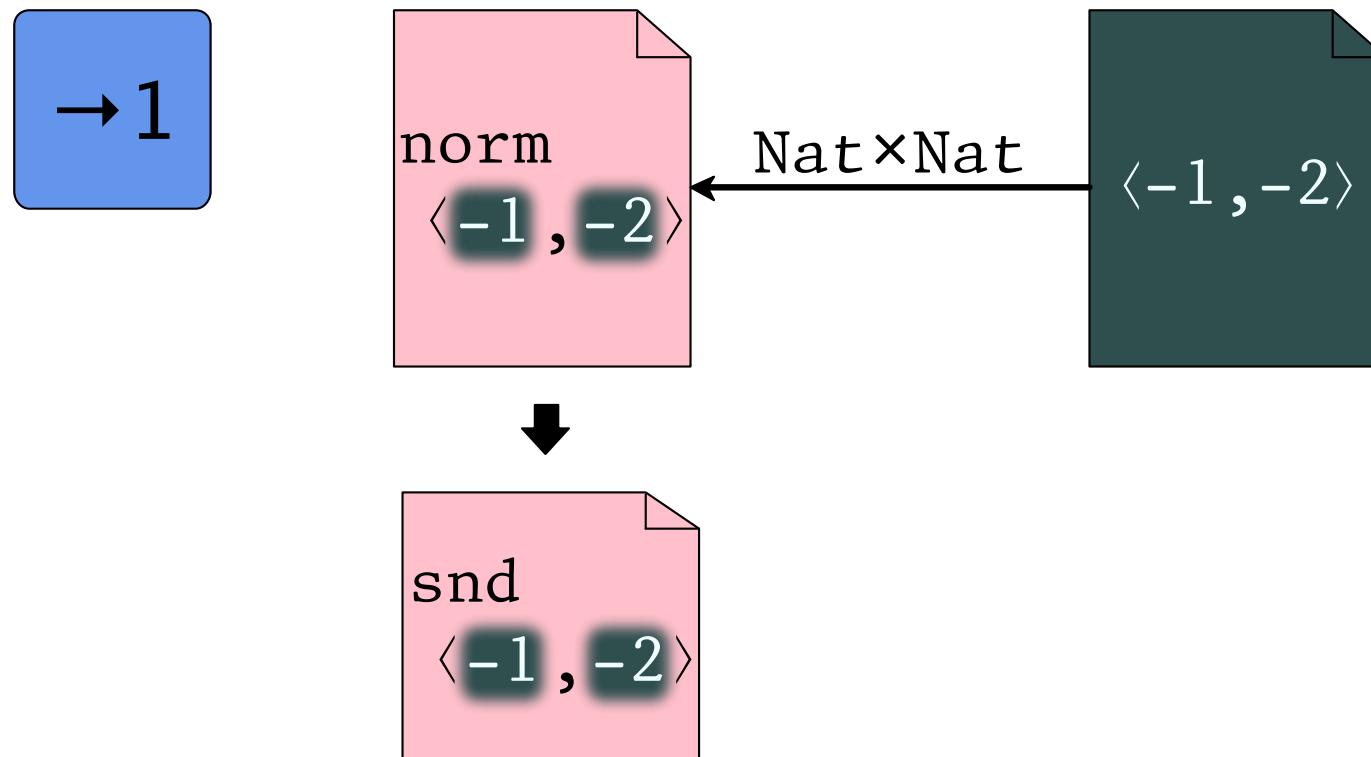
first-order (enforce type constructors)



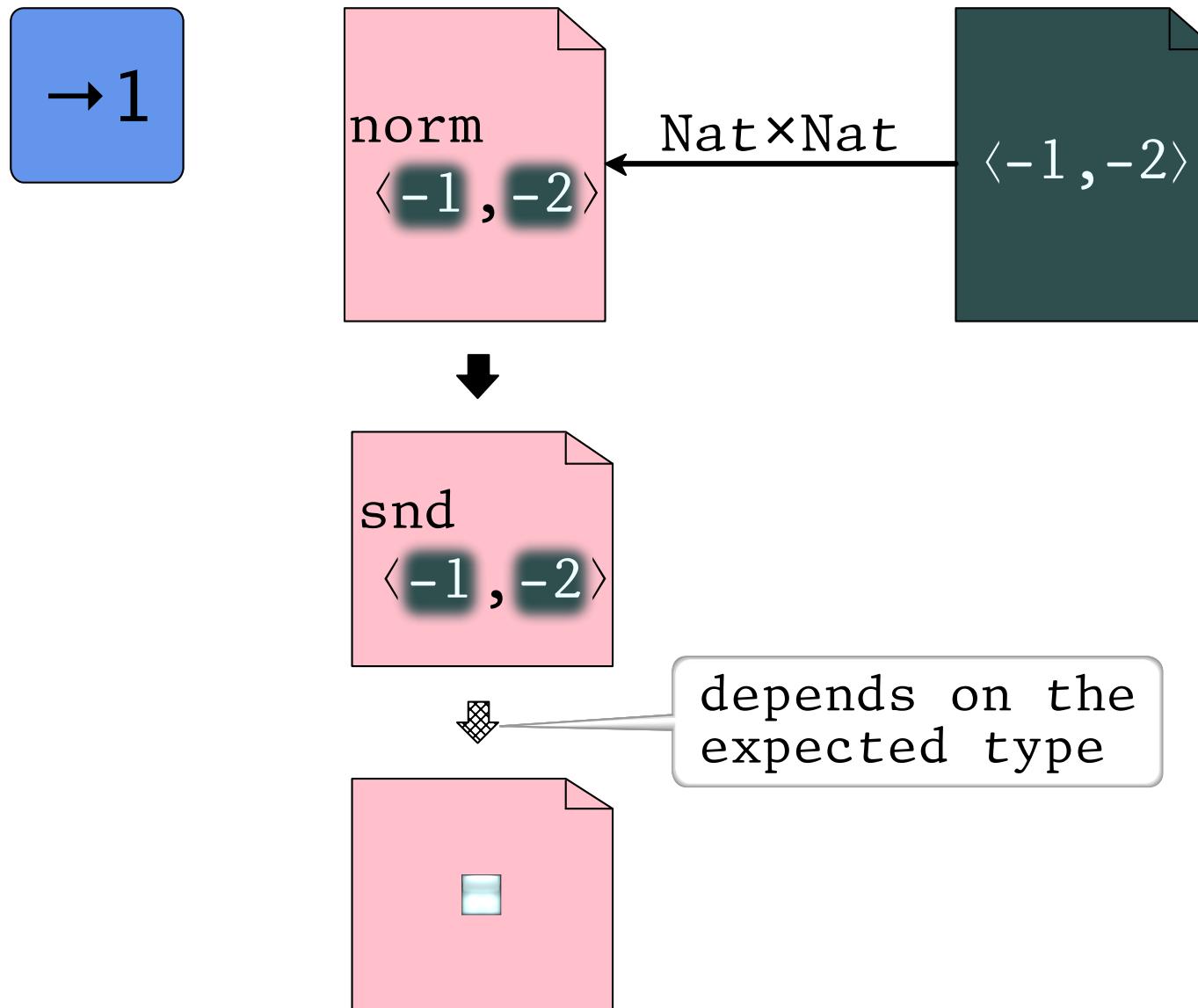
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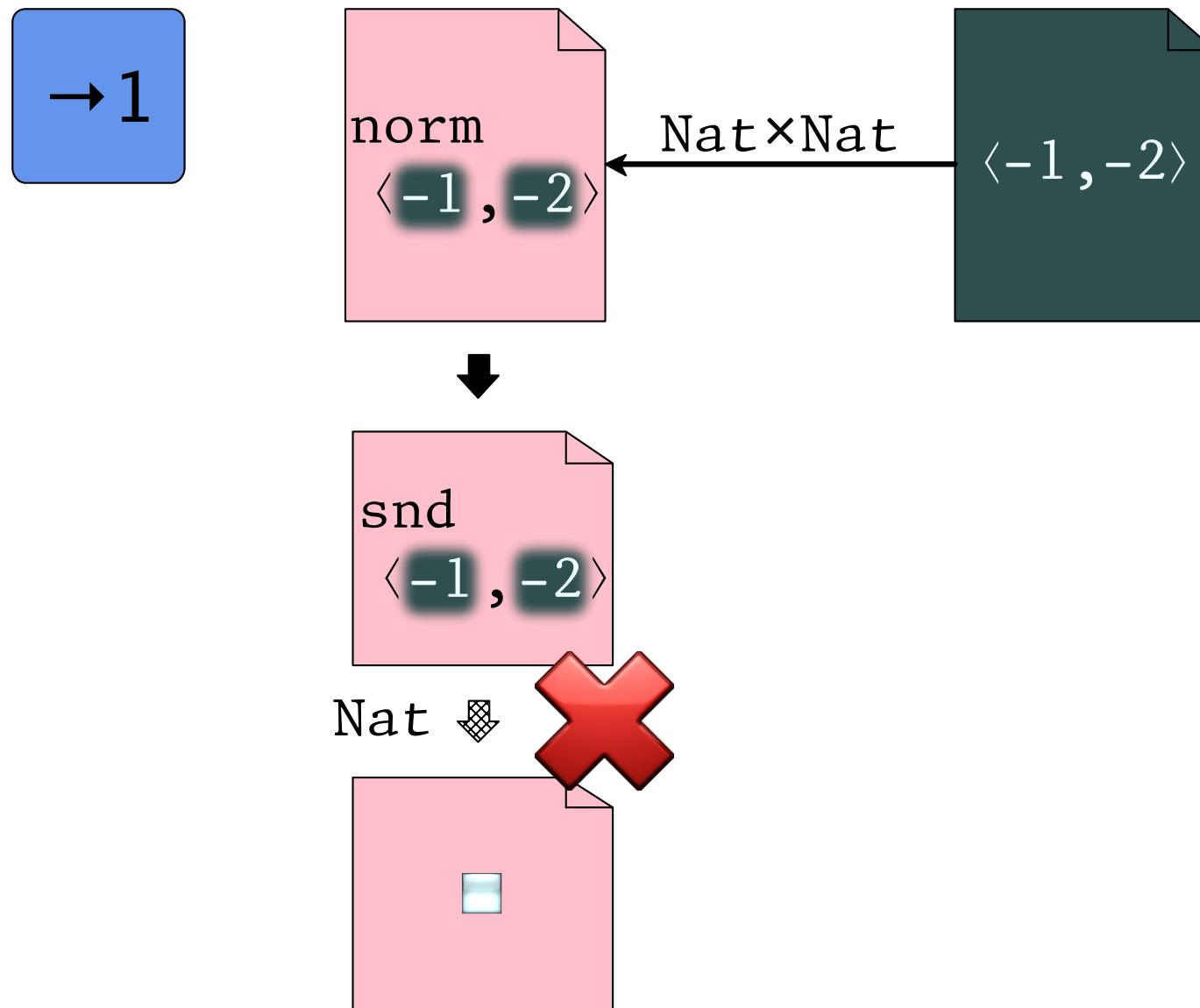
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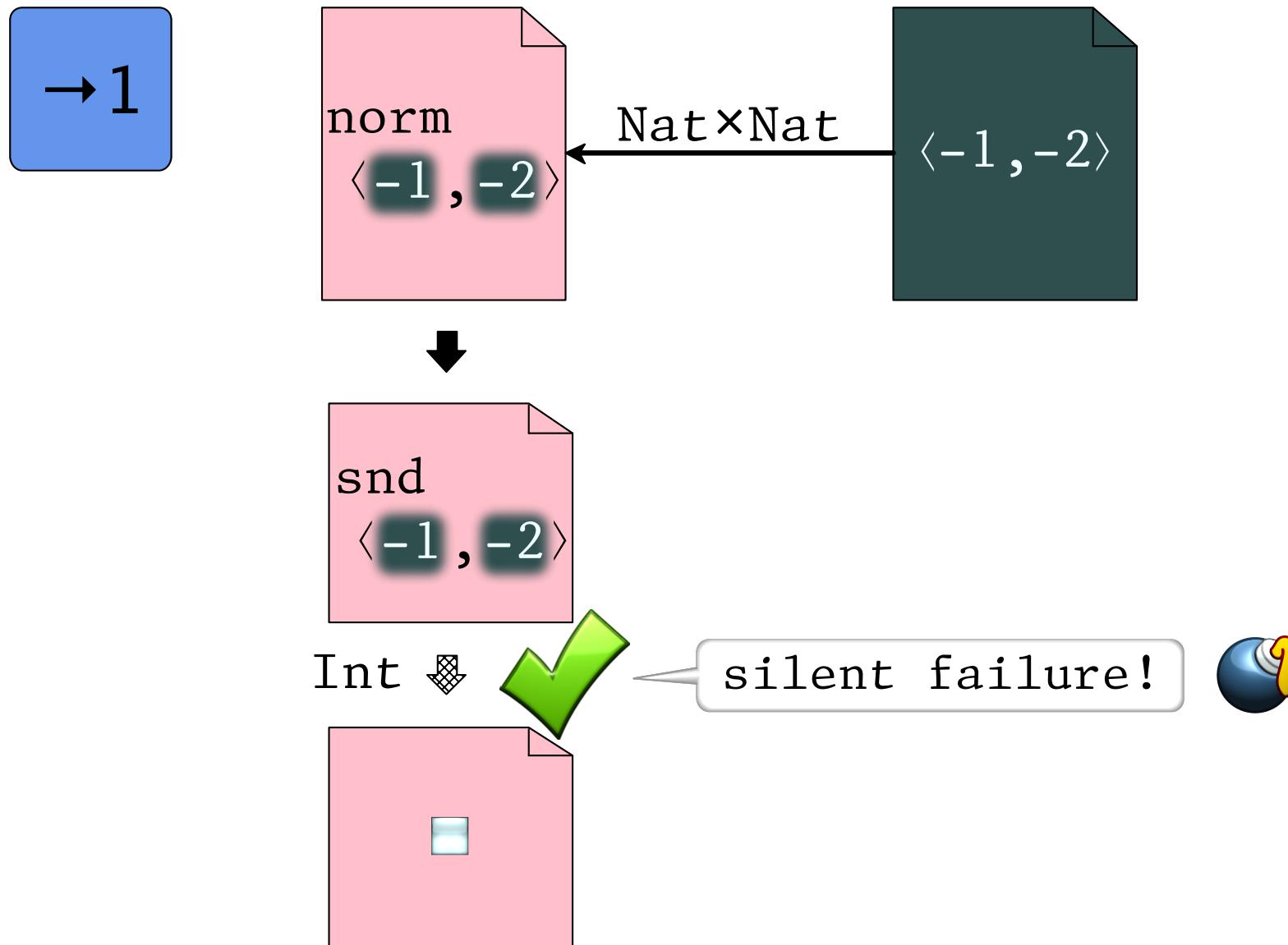
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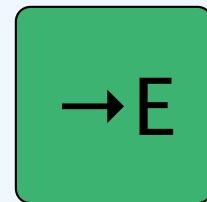
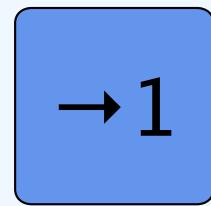
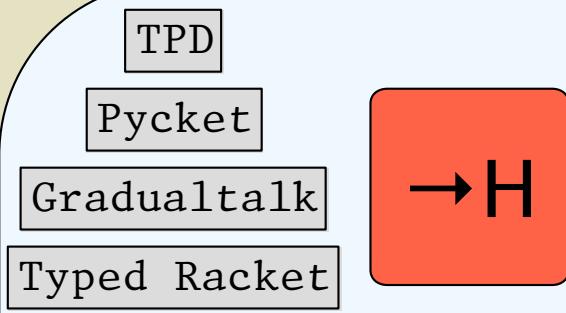


first-order (enforce type constructors)

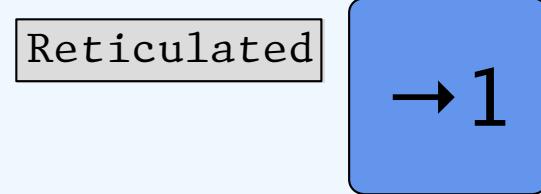
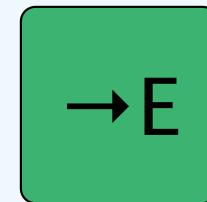
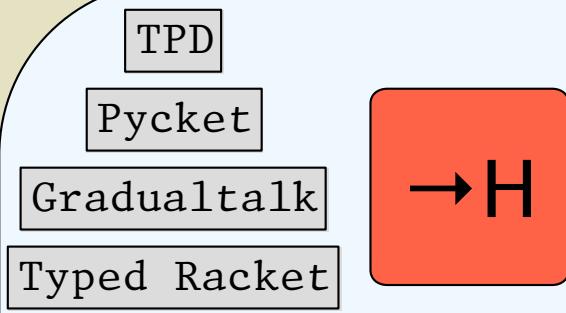


first-order (enforce type constructors)



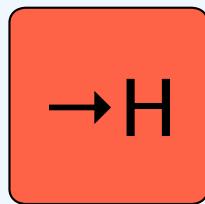


(the systems landscape)

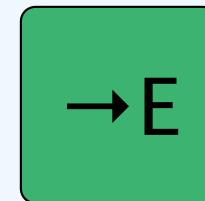
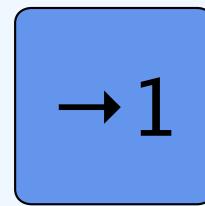


(the systems landscape)

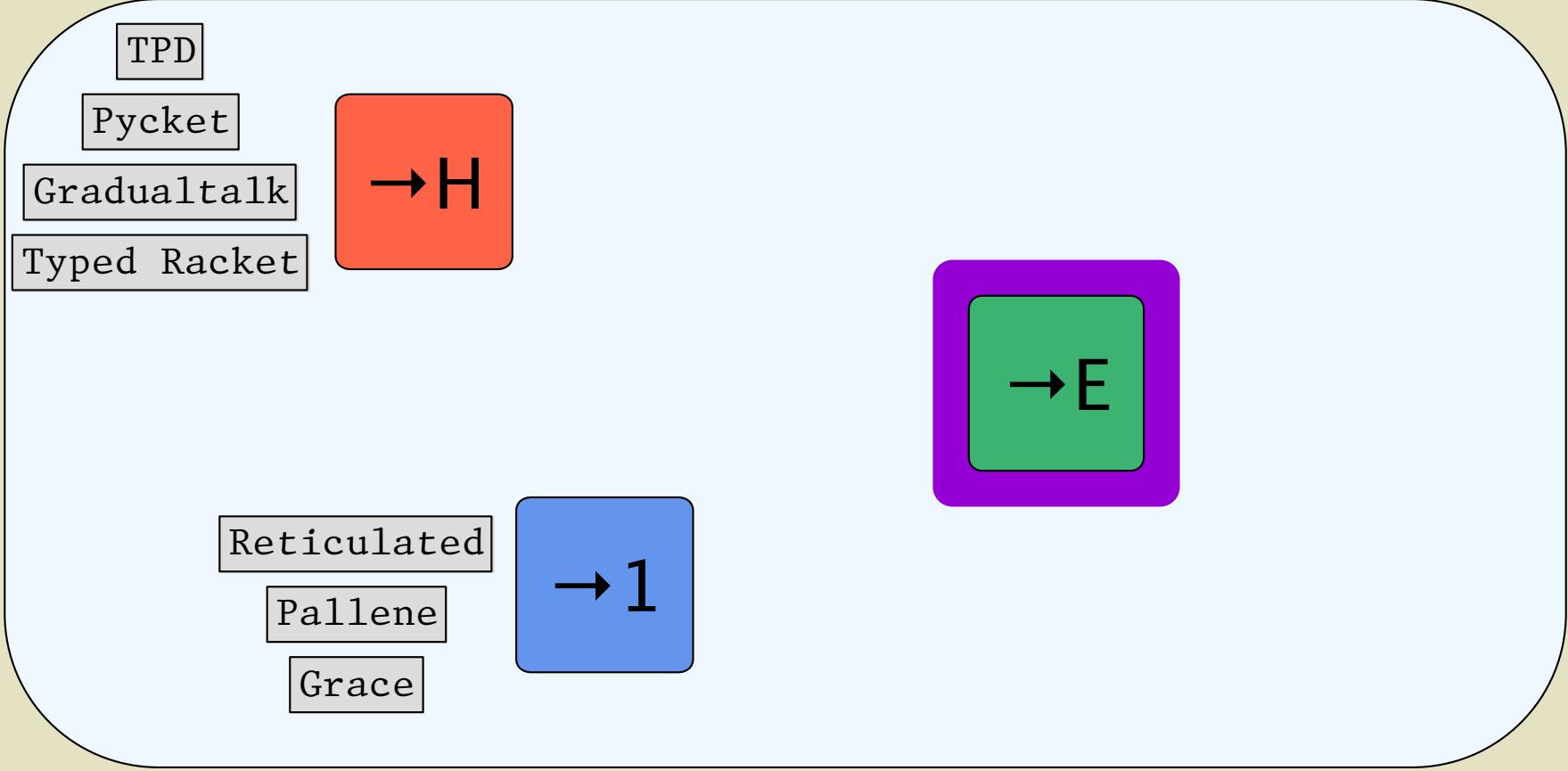
TPD
Pycket
Gradualtalk
Typed Racket



Reticulated
Pallene
Grace

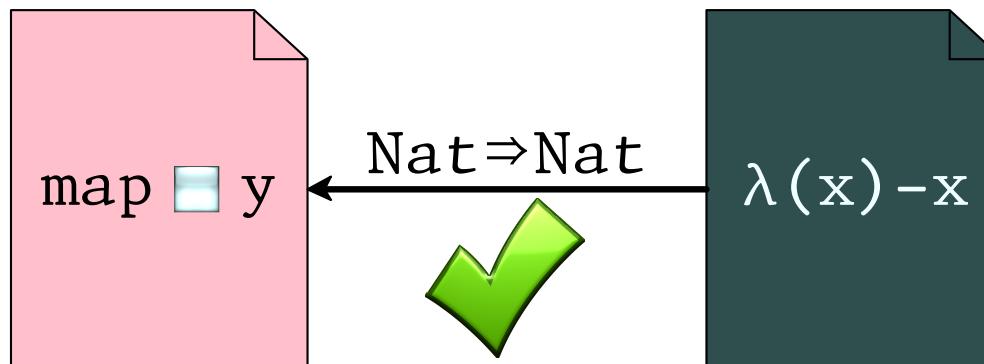
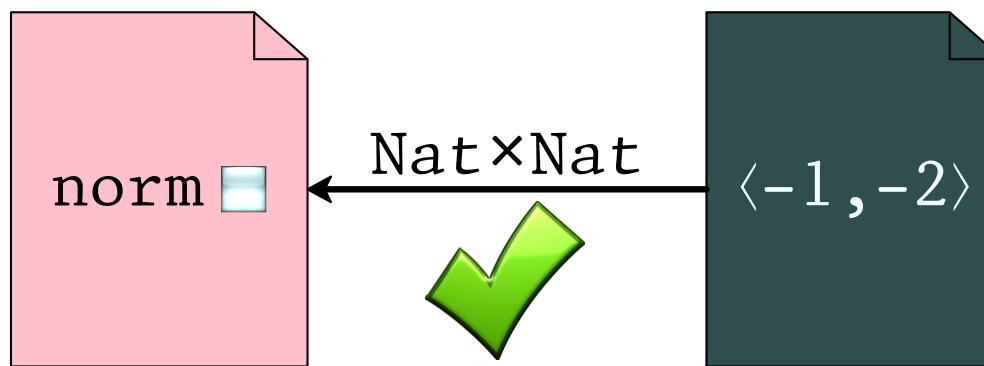
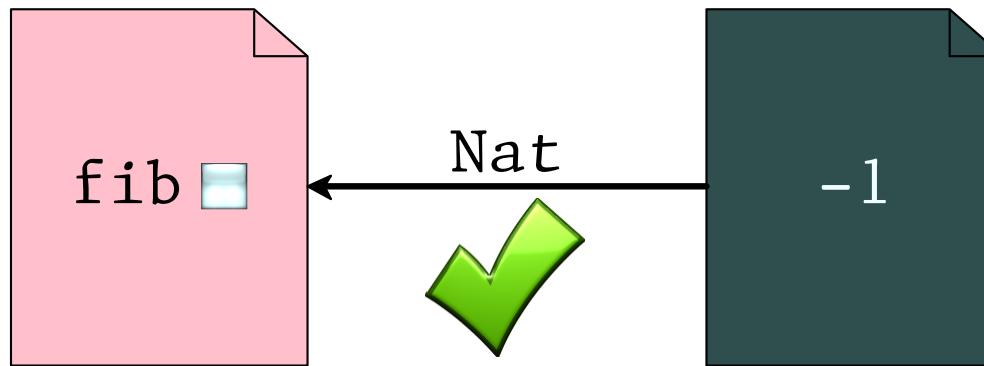
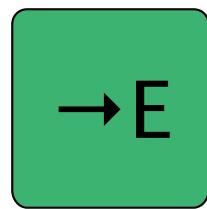


(the systems landscape)

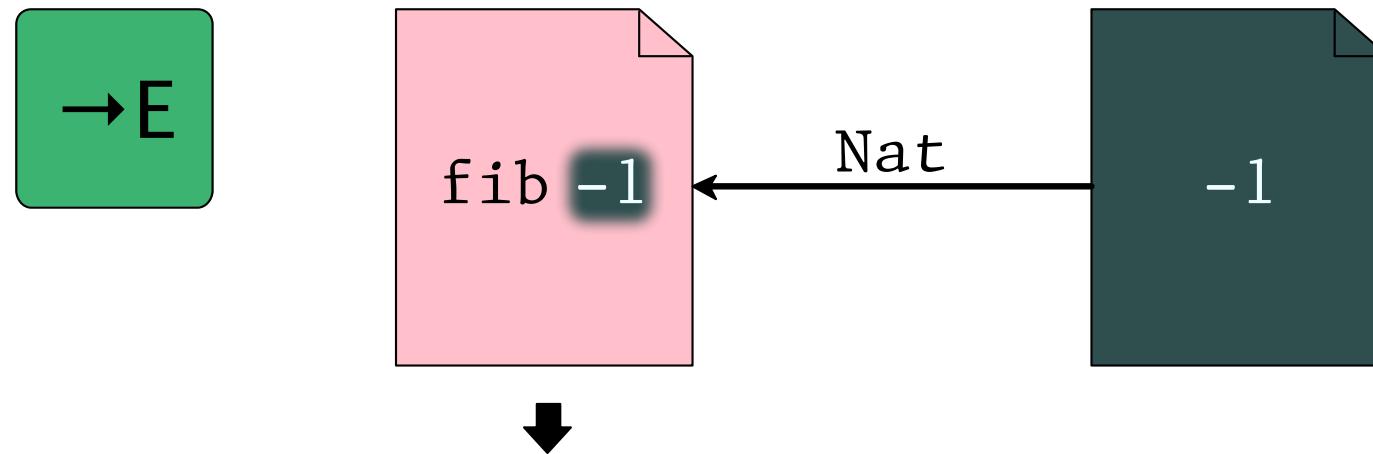


(the systems landscape)

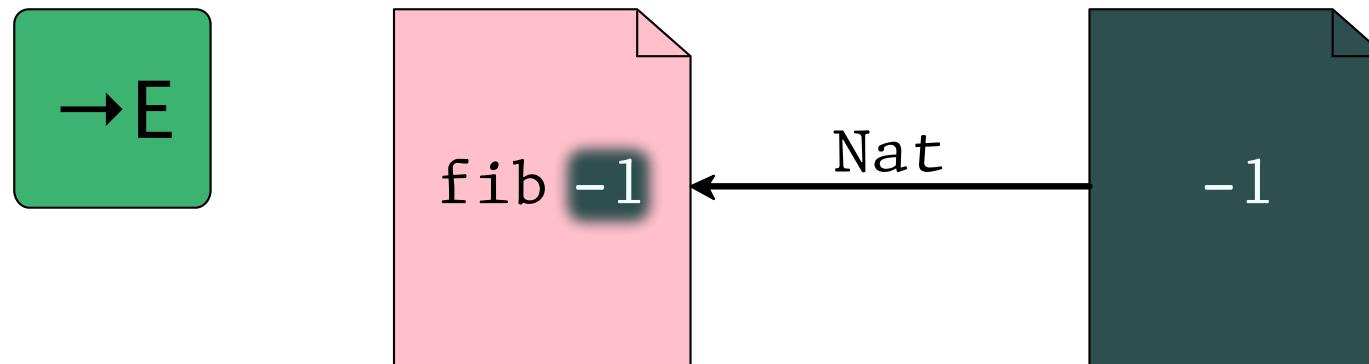
erasure (ignore types)



erasure (ignore types)



erasure (ignore types)



error?

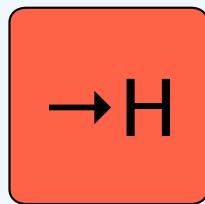
diverges?

0

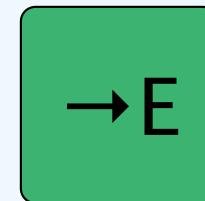
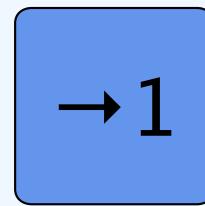
???



TPD
Pycket
Gradualtalk
Typed Racket

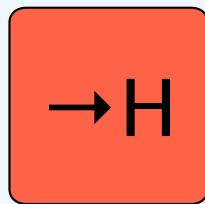


Reticulated
Pallene
Grace

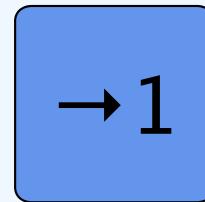


(the systems landscape)

TPD
Pycket
Gradualtalk
Typed Racket



Reticulated
Pallene
Grace



mypy

Flow

Hack

Pyre

Pytype

rtc

MACLISP

Common Lisp

Strongtalk

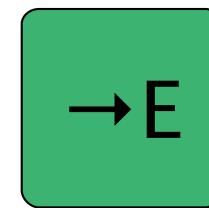
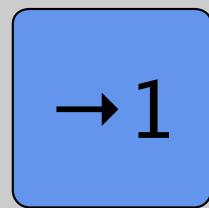
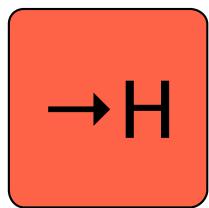
TypeScript

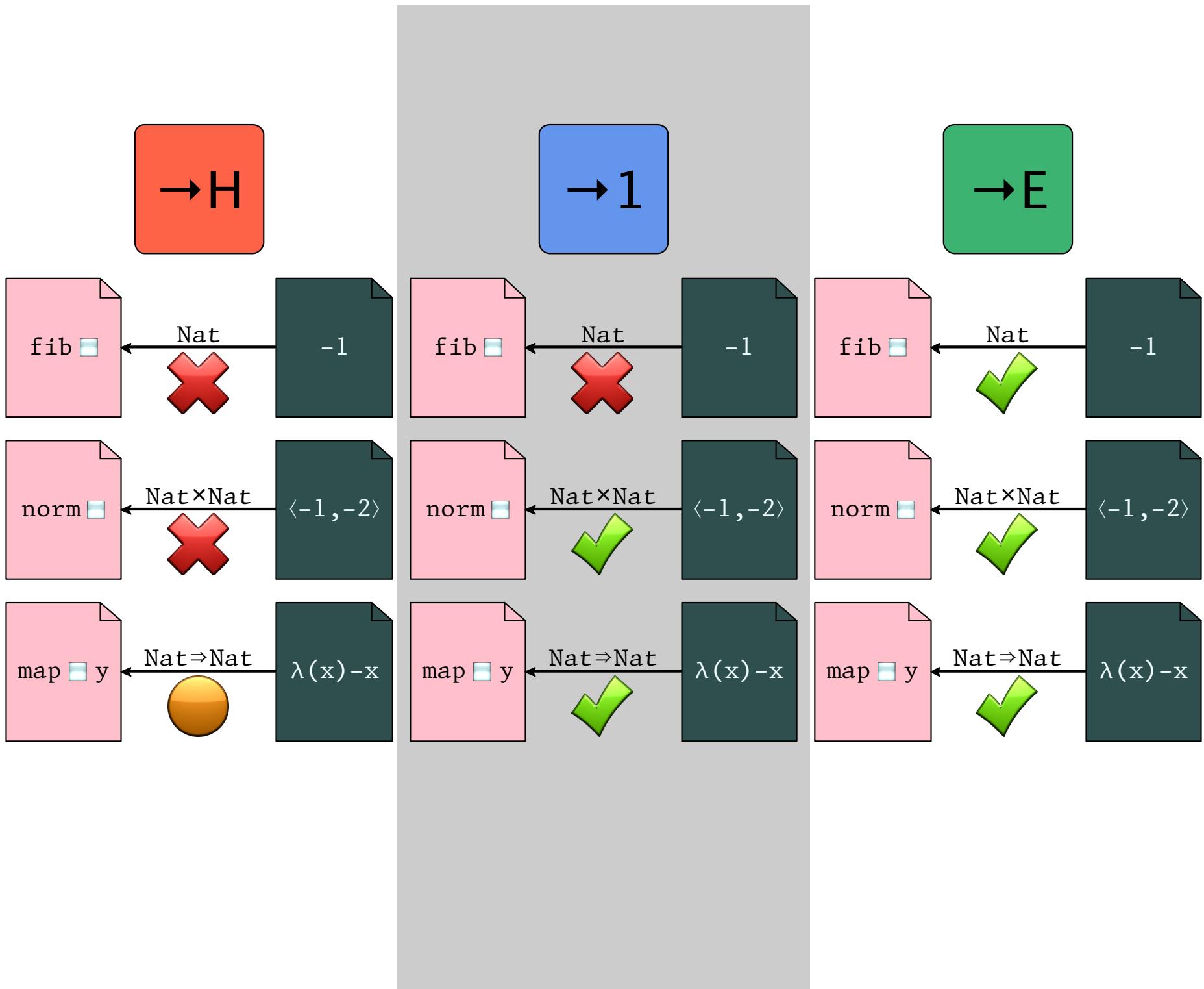
Typed Clojure

Typed Lua

Dart 1

(the systems landscape)







Type violations discovered



Type violations discovered



Type violations discovered

Theorem (\supseteq) :

- if $e \xrightarrow{\rightarrow 1} \text{Error}$
then $e \xrightarrow{\rightarrow H} \text{Error}$

- if $e \xrightarrow{\rightarrow E} \text{Error}$
then $e \xrightarrow{\rightarrow 1} \text{Error}$



Type violations discovered

Theorem (\supseteq) :

- if $e \rightarrow_1 \text{Error}$
- then $e \rightarrow_H \text{Error}$

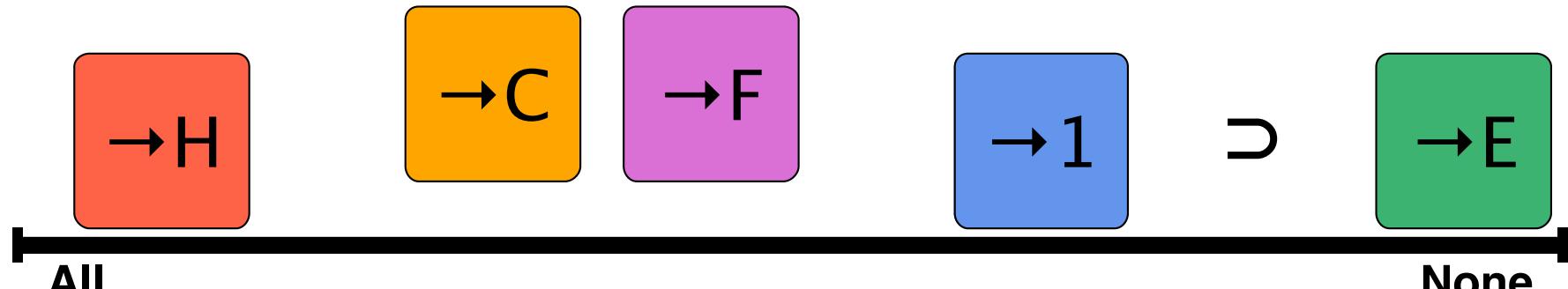
- if $e \rightarrow_E \text{Error}$
- then $e \rightarrow_1 \text{Error}$

Counterexamples (\nexists) :

- see prev. slide

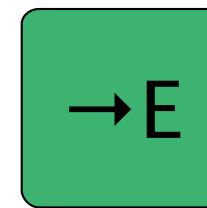
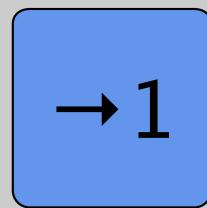
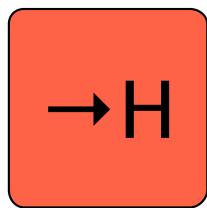


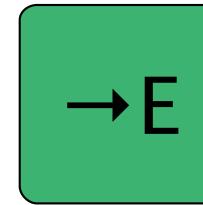
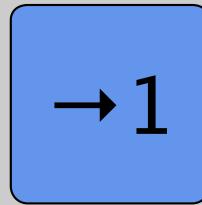
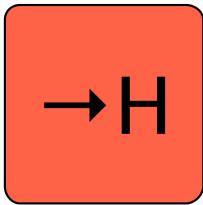
Type violations discovered



Type violations discovered

Appendix: two other semantics

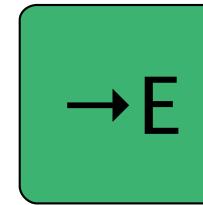
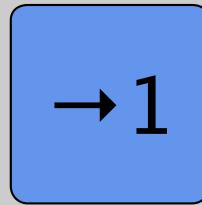
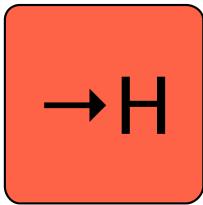




Type Soundness (simplified) :

if $\vdash e : T$ then either:

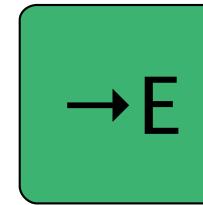
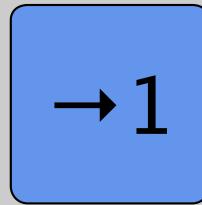
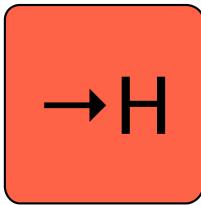
- $e \rightarrow^* v$ and $\vdash v : T$
- e diverges
- $e \rightarrow^* \text{Error}$



Type Soundness (simplified) :

if $\vdash e : T$ then either:

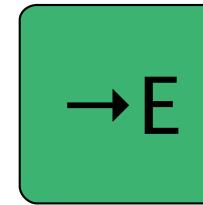
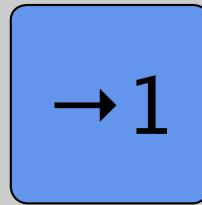
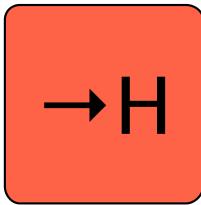
- $e \rightarrow^* v$ and $\vdash v : T$
- e diverges
- $e \rightarrow^* \text{Error}$



Type Soundness (simplified) :

if $\vdash e : T$ then either:

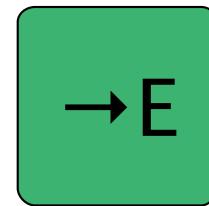
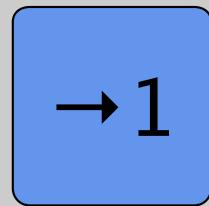
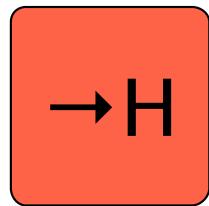
- $e \rightarrow^* v$ and $\vdash v : T$
- e diverges
- $e \rightarrow^* \text{Error}$



Type Soundness (simplified) :

if $\vdash e : T$ then either:

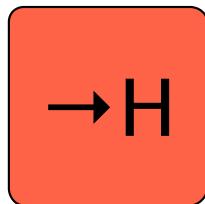
- $e \rightarrow^* v$ and $\vdash v : T$
 - e diverges
 - $e \rightarrow^* \text{Error}$
-



→H Soundness:

if $\vdash e : \tau$ then either:

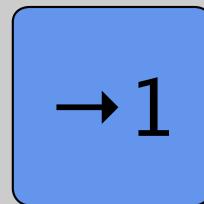
- $e \rightarrow^* v$ and $\vdash v : \tau$
- e diverges
- $e \rightarrow^* \text{Error}$



→H Soundness:

if $\vdash e : \tau$ then either:

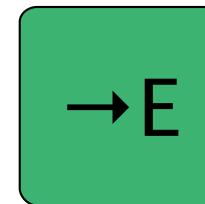
- $e \rightarrow^* v$ and $\vdash v : \tau$
- e diverges
- $e \rightarrow^* \text{Error}$

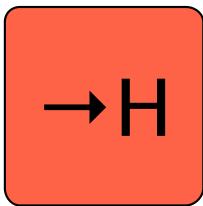


→1 Soundness:

if $\vdash e : \tau$ then either:

- $e \rightarrow^* v$ and $\vdash v : C(\tau)$
- e diverges
- $e \rightarrow^* \text{Error}$

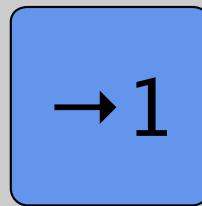




→H Soundness:

if $\vdash e : \tau$ then either:

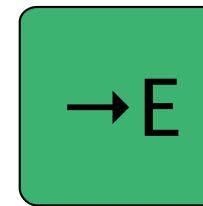
- $e \rightarrow^* v$ and $\vdash v : \tau$
- e diverges
- $e \rightarrow^* \text{Error}$



→1 Soundness:

if $\vdash e : \tau$ then either:

- $e \rightarrow^* v$ and $\vdash v : C(\tau)$
- e diverges
- $e \rightarrow^* \text{Error}$



→E Soundness:

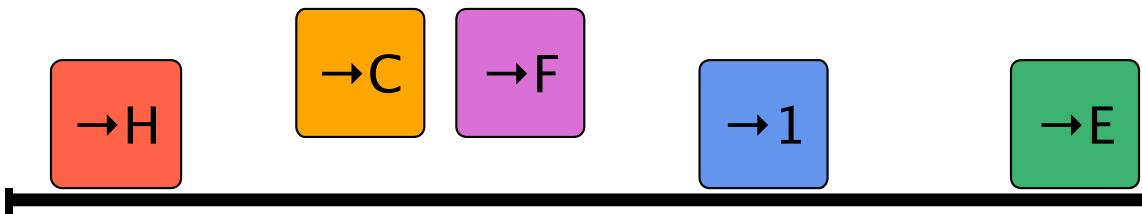
if $\vdash e : \tau$ then either:

- $e \rightarrow^* v$ and $\vdash v$
- e diverges
- $e \rightarrow^* \text{Error}$

Is type soundness all-or-nothing?

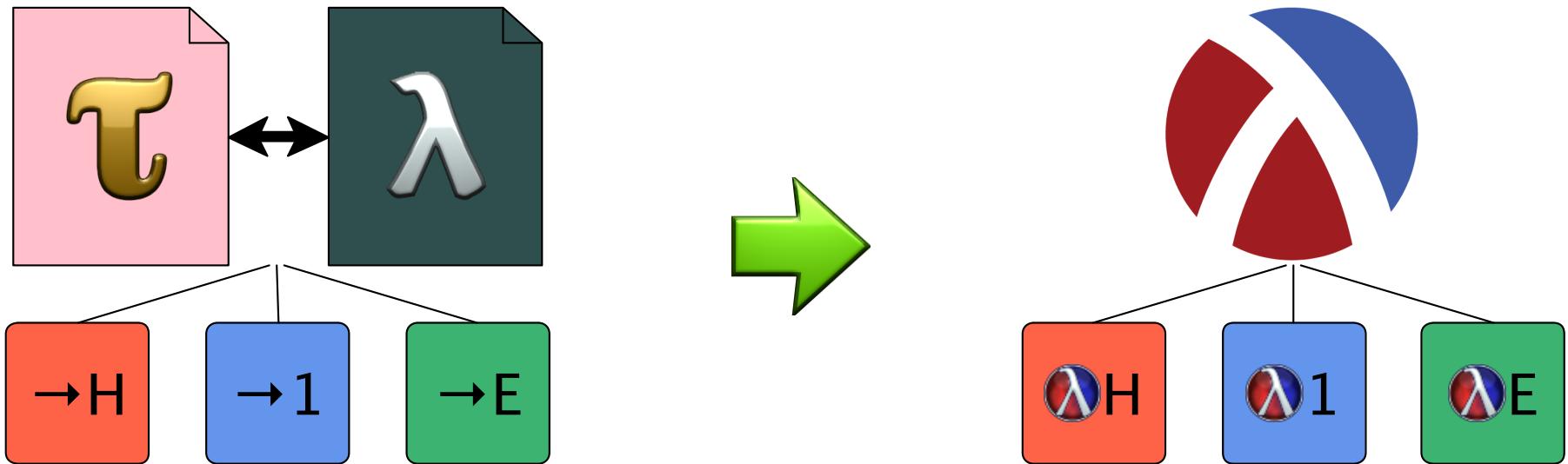
Is type soundness all-or-nothing?

No! (in a mixed-typed language)



Implementation

How does type soundness affect performance?

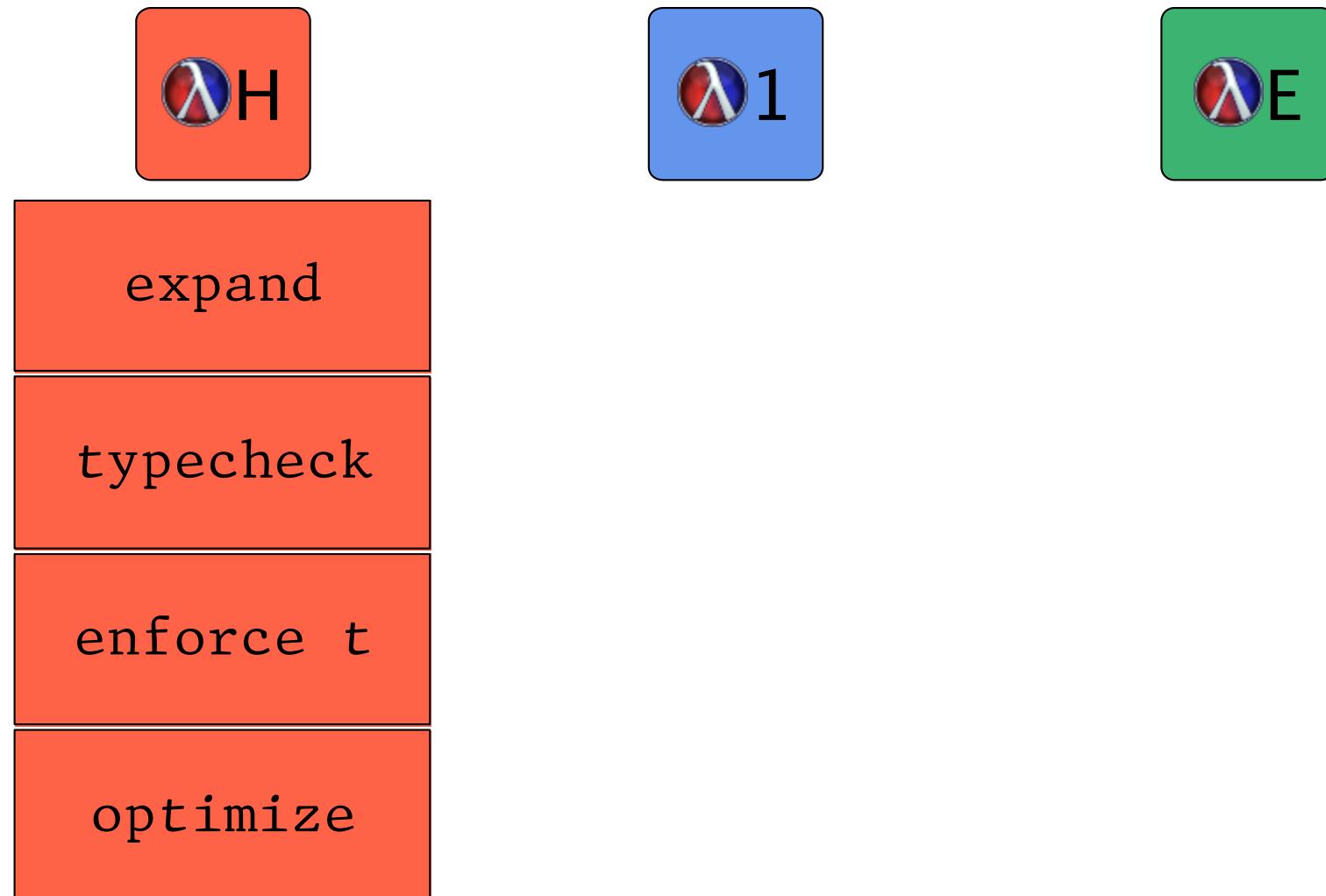


model => implementation

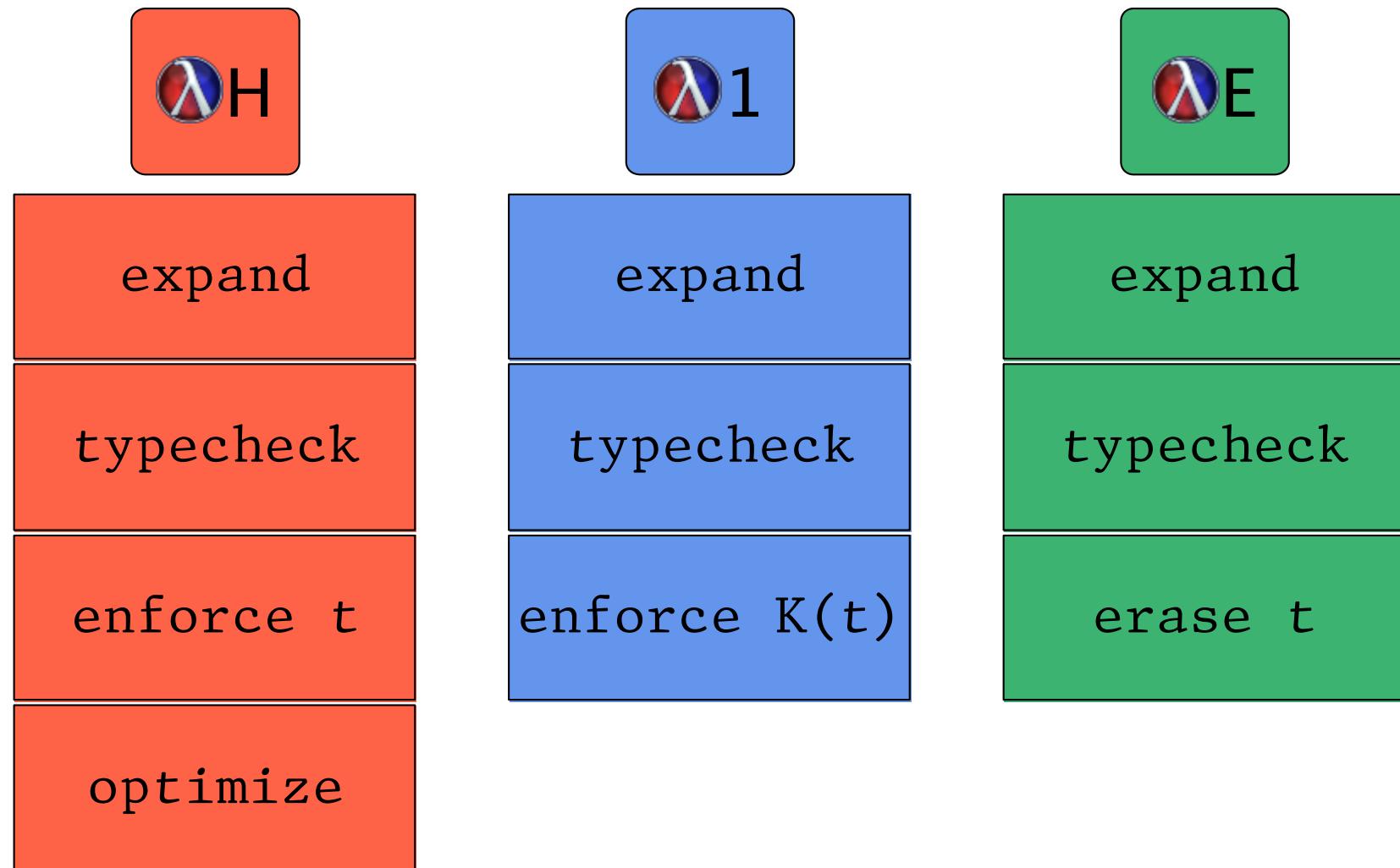
3 Compilers



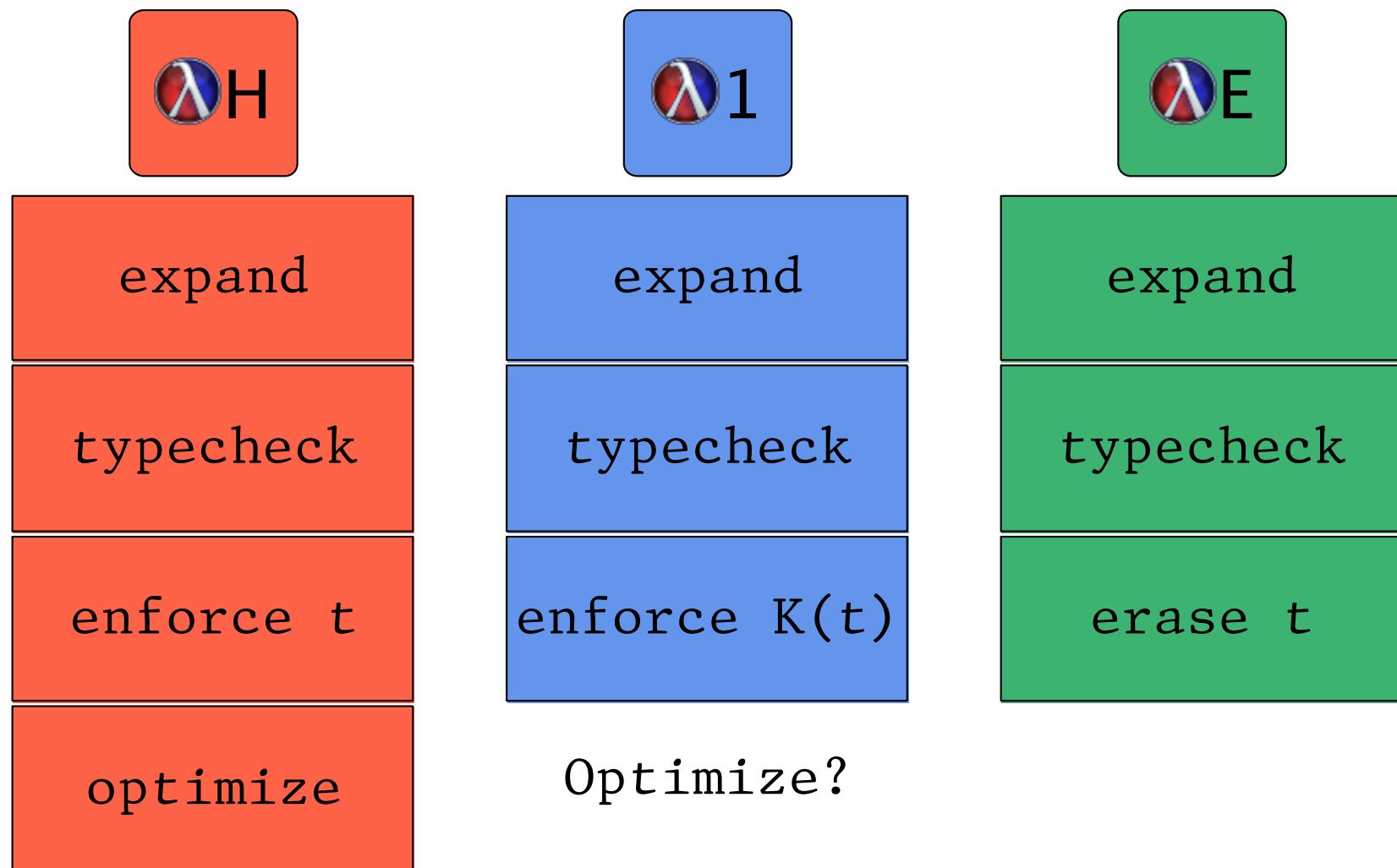
3 Compilers



3 Compilers



3 Compilers

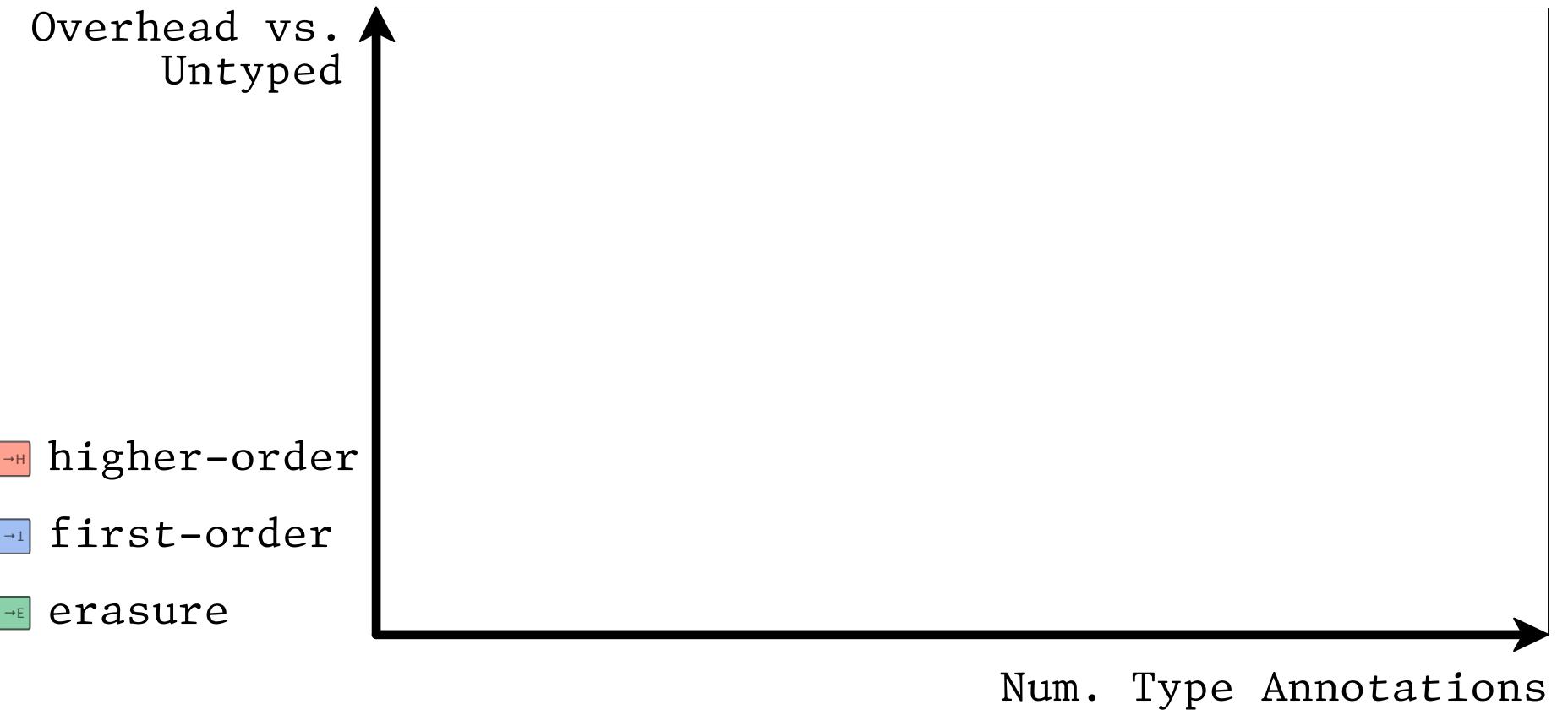


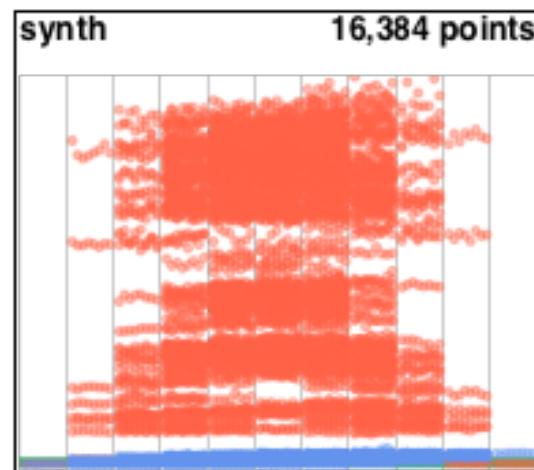
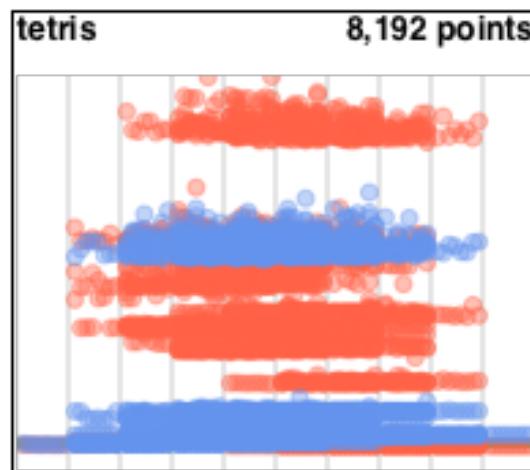
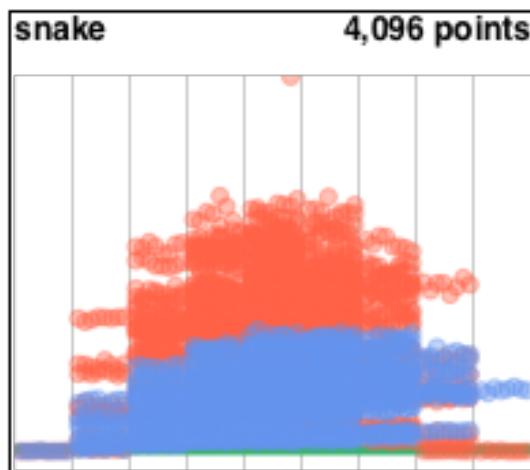
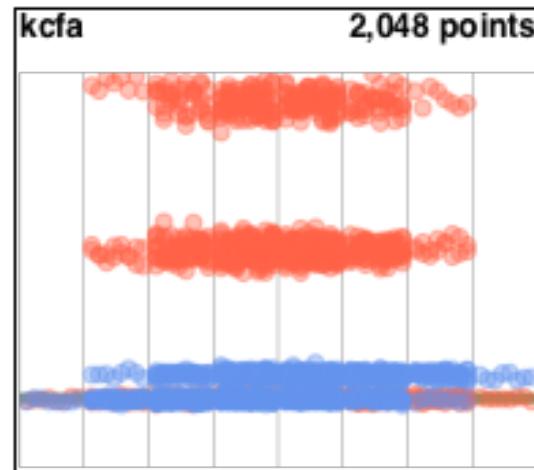
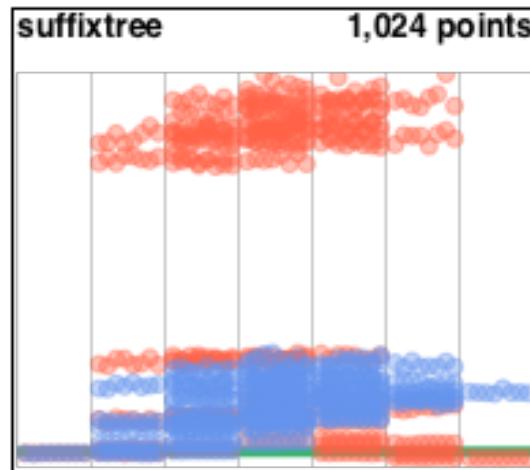
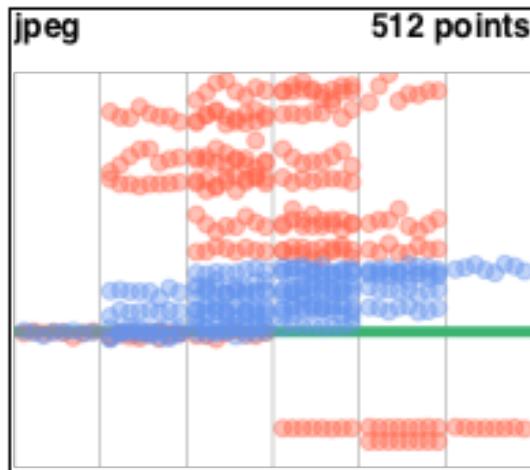
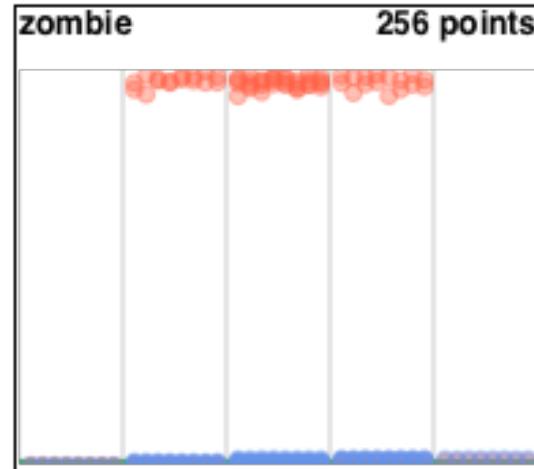
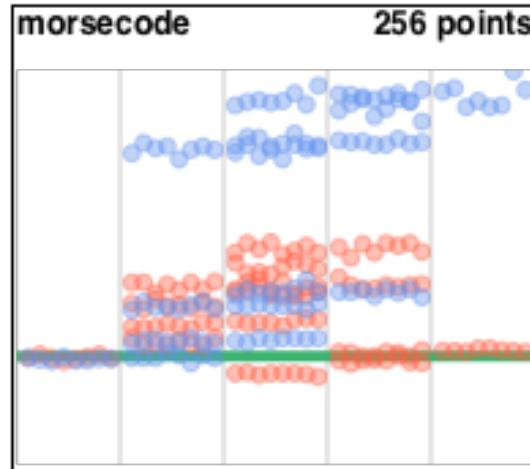
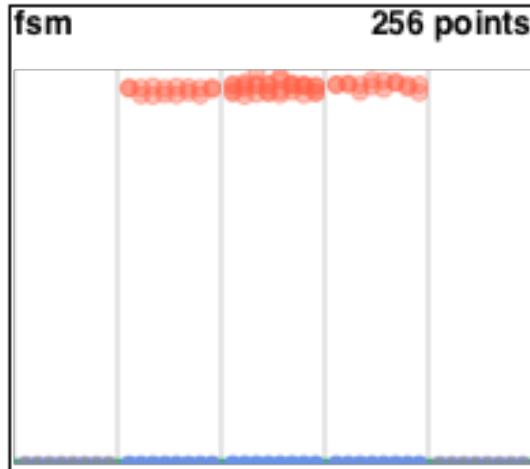
Experiment (method from POPL'16)

- 10 benchmark programs
- 2 to 10 modules each
- 4 to 1024 configurations each
- compare overhead to untyped

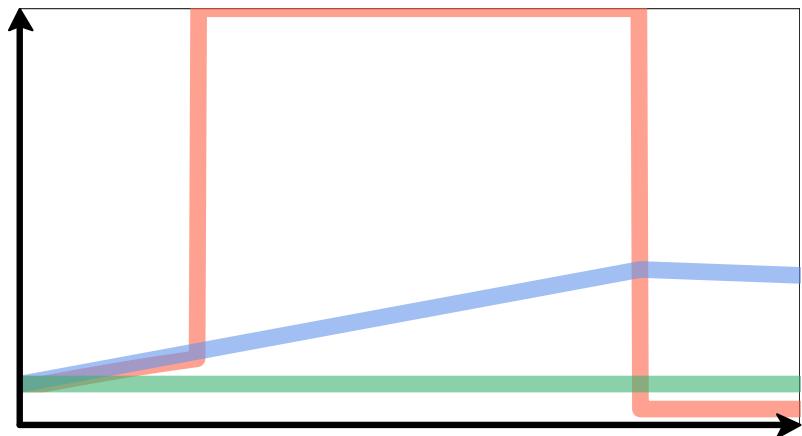
docs.racket-lang.org/gtp-benchmarks

Soundness vs. Performance

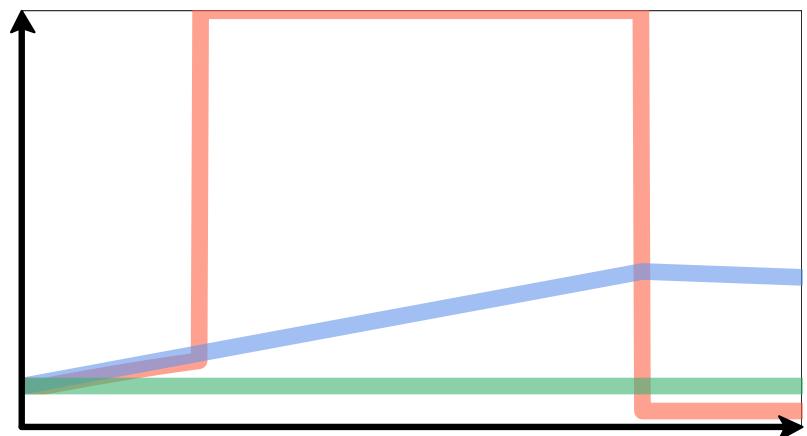




Performance Implications

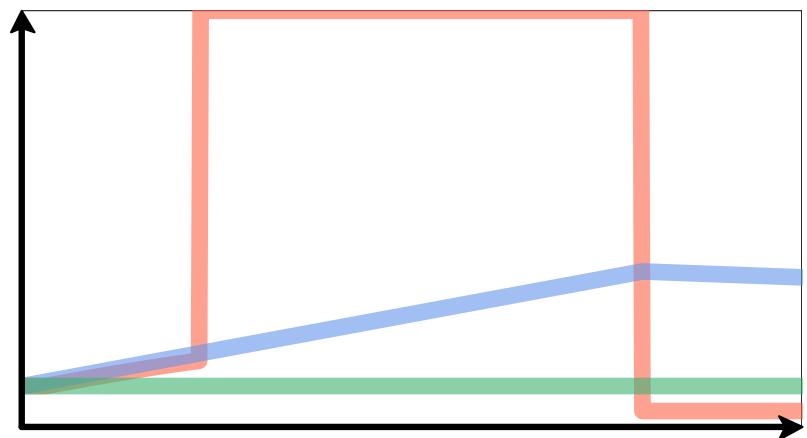


Performance Implications

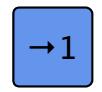


→H add types to remove all critical boundaries

Performance Implications

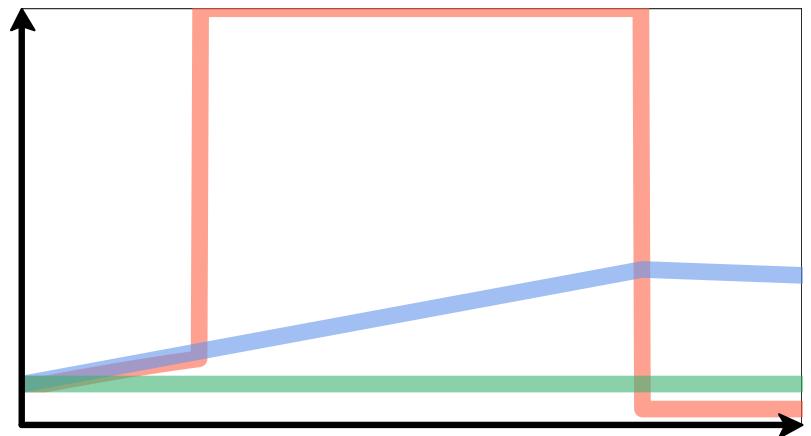


add types to remove all critical boundaries



add types sparingly

Performance Implications



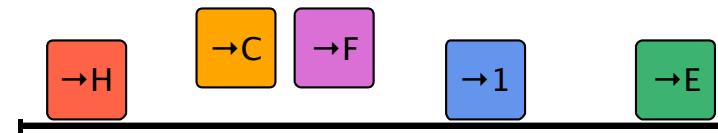
- H add types to remove all critical boundaries
- I add types sparingly
- E add types anywhere, doesn't matter

Takeaways

Takeaways

Theorists:

type soundness is NOT
all-or-nothing



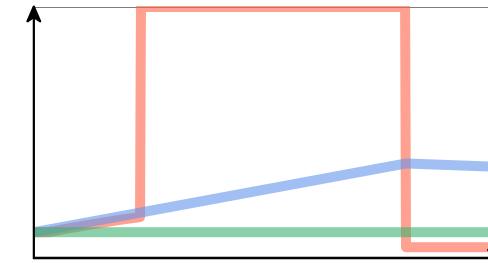
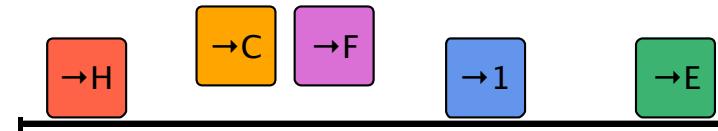
Takeaways

Theorists:

type soundness is NOT
all-or-nothing

Implementors:

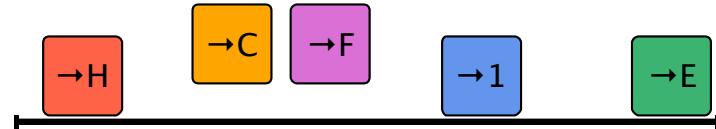
can we change the
performance landscape?



Takeaways

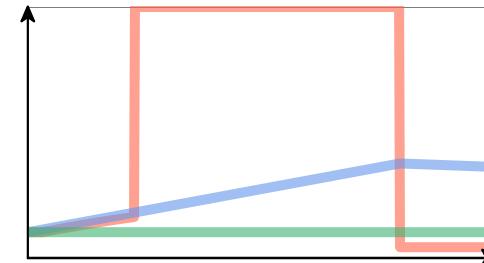
Theorists:

type soundness is NOT
all-or-nothing



Implementors:

can we change the
performance landscape?

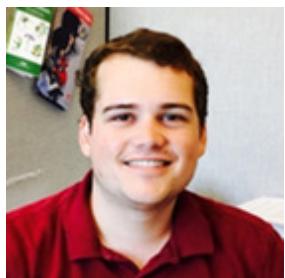
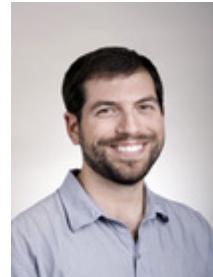


Users:

soundness affects **run-time**
and **debug-time**



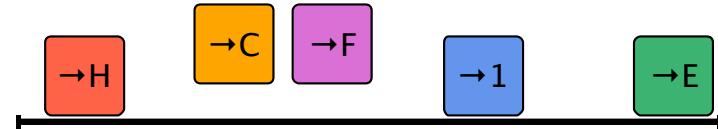
Special Thanks



Takeaways

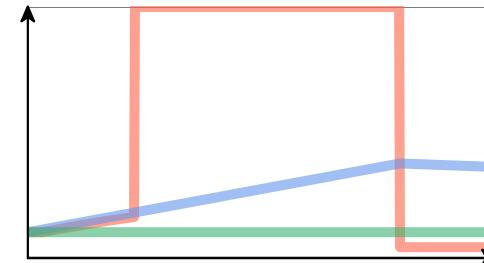
Theorists:

type soundness is NOT
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Implementors:

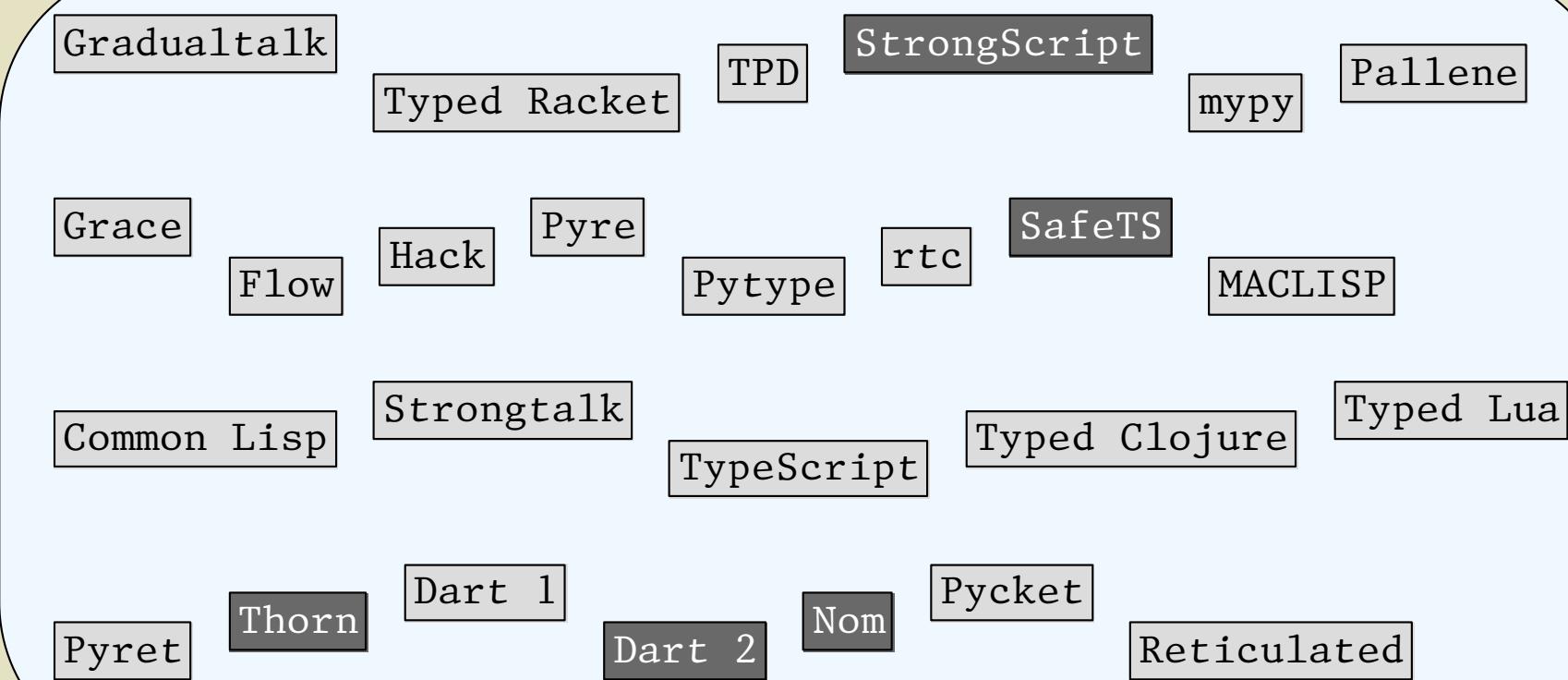
can we change the
performance landscape?



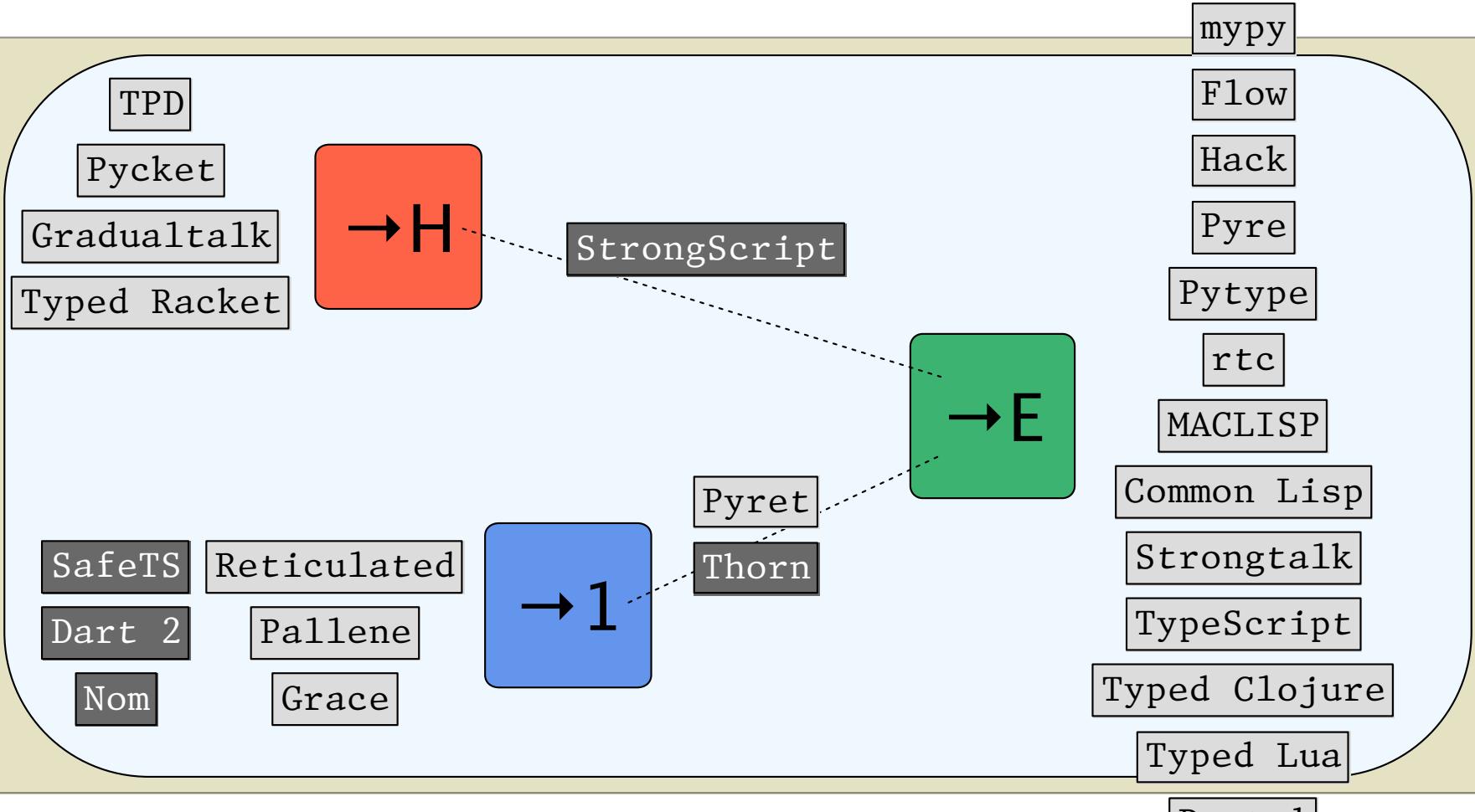
Users:

soundness affects **run-time**
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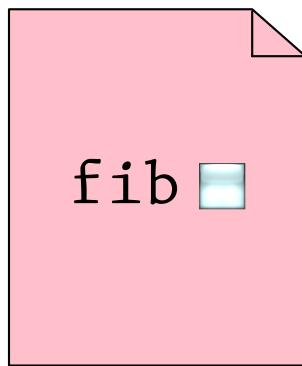
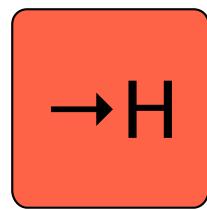


(the systems landscape)

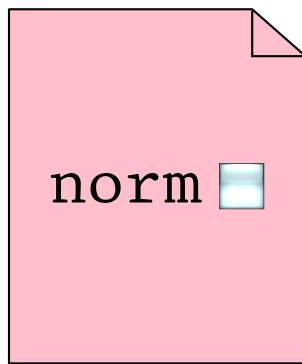
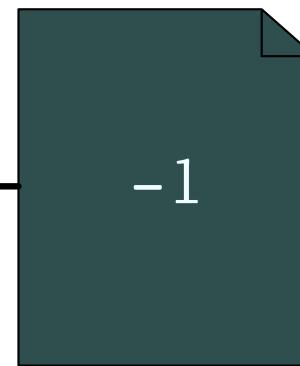


(the systems landscape)

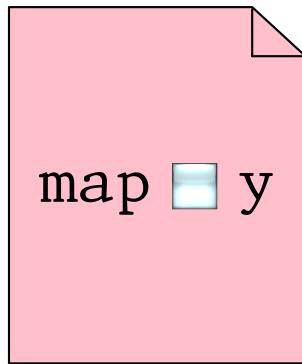
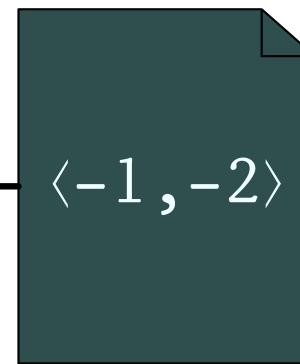
higher-order (enforce full types)



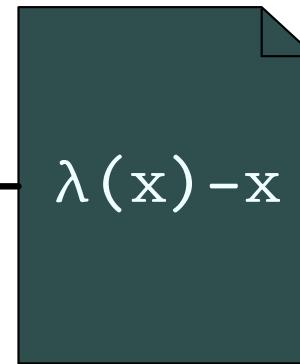
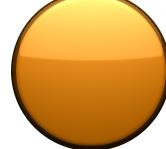
Nat



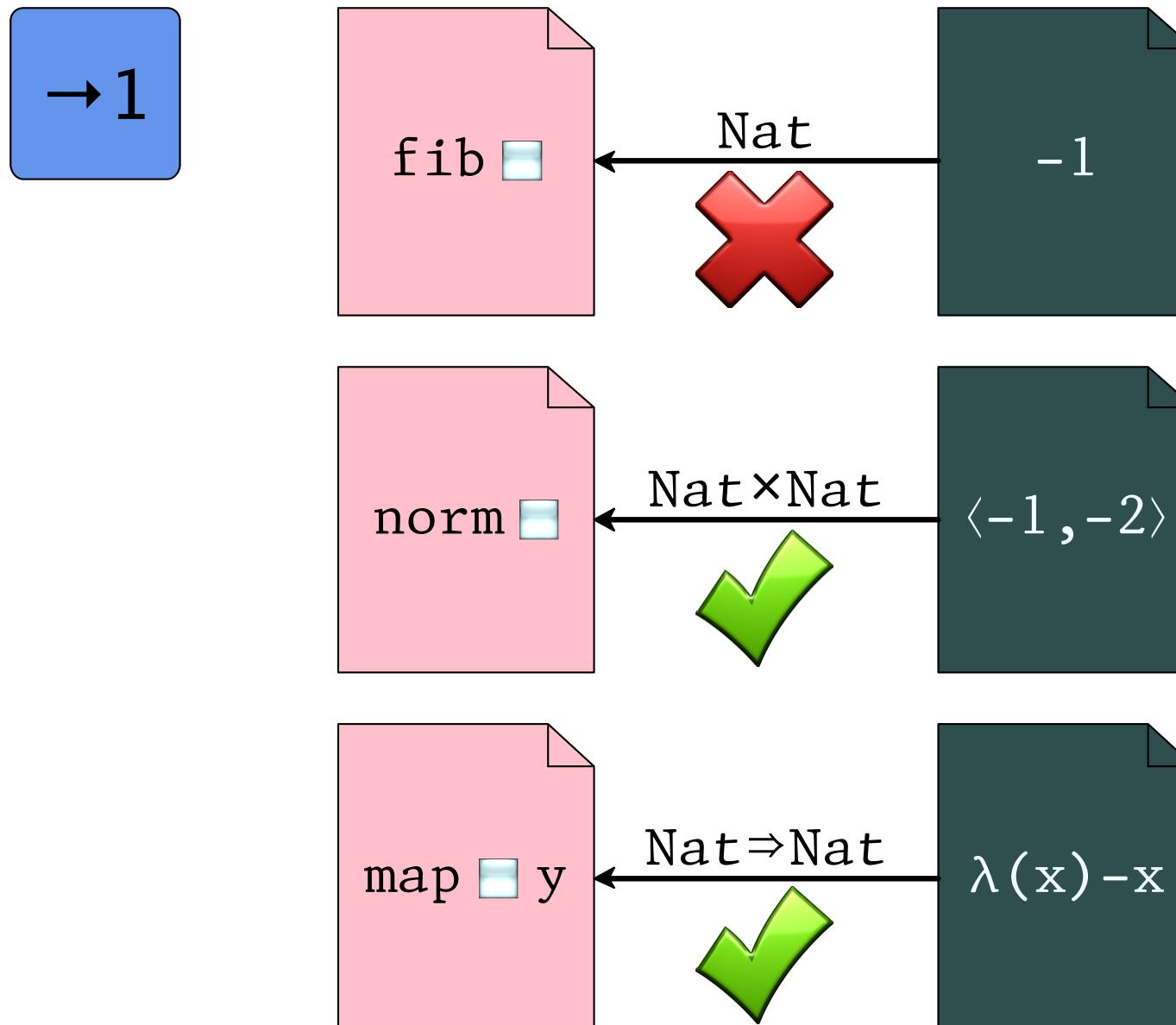
Nat × Nat



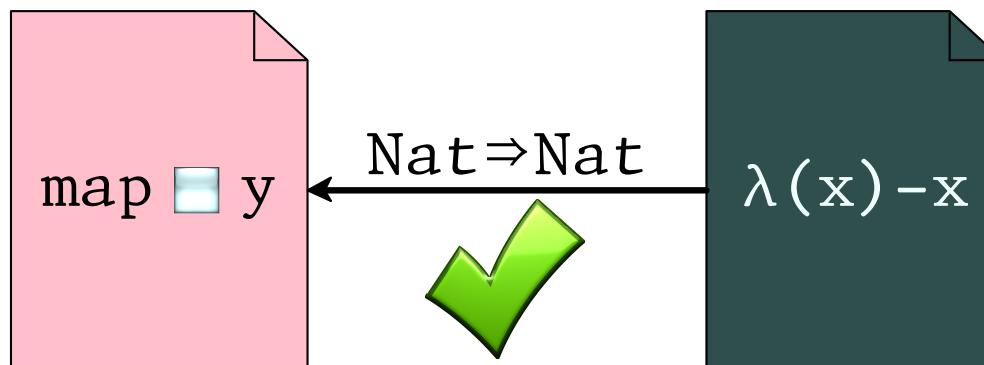
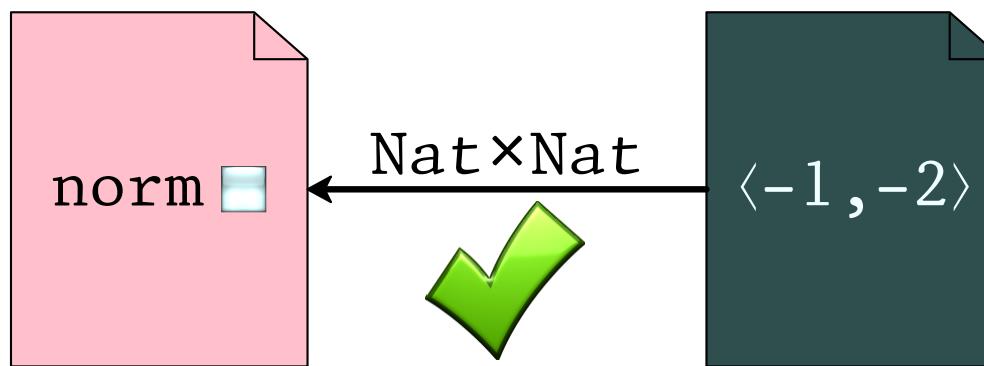
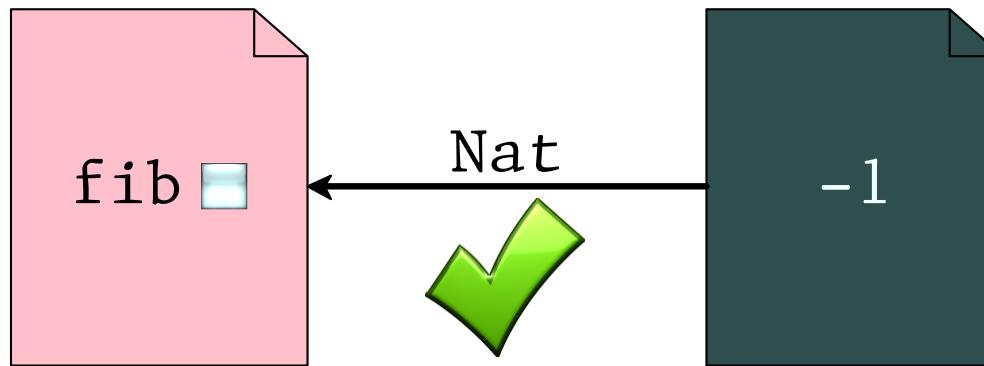
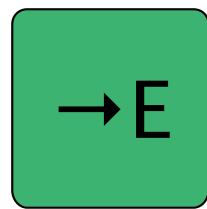
Nat → Nat

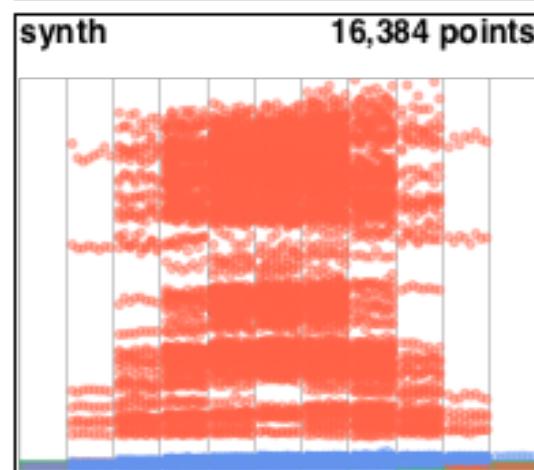
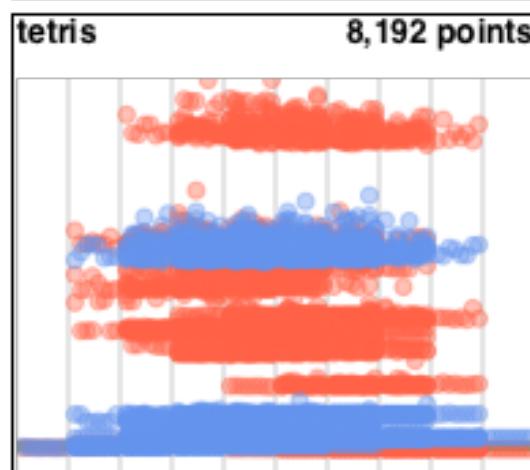
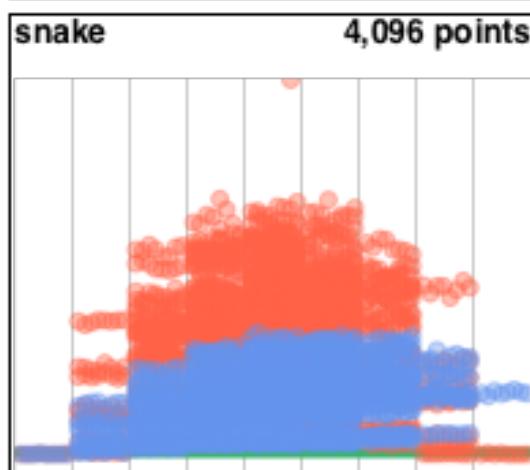
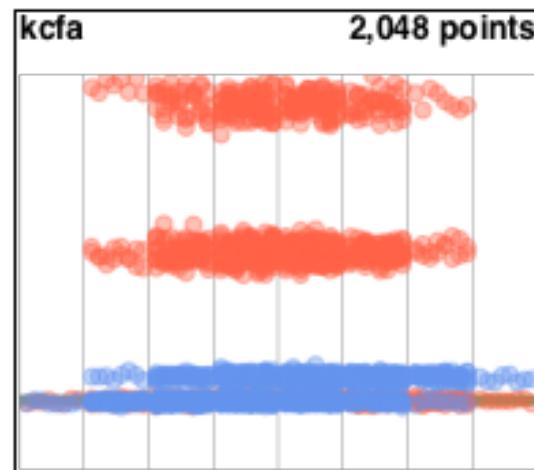
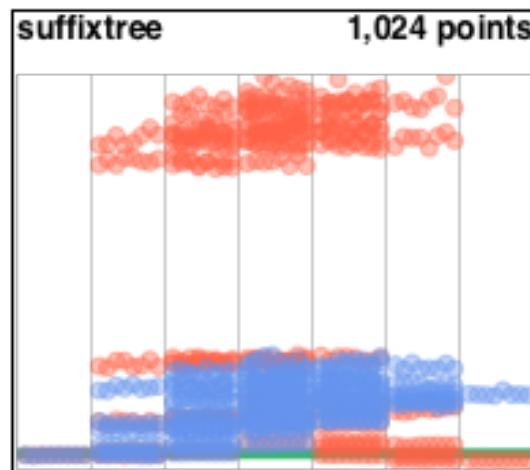
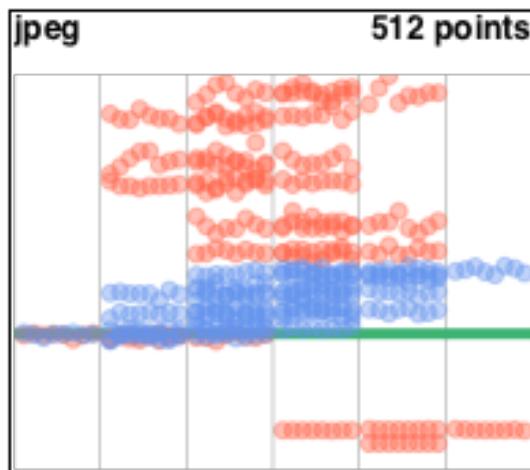
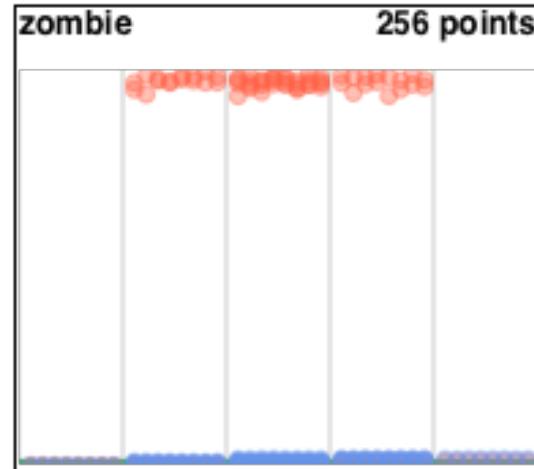
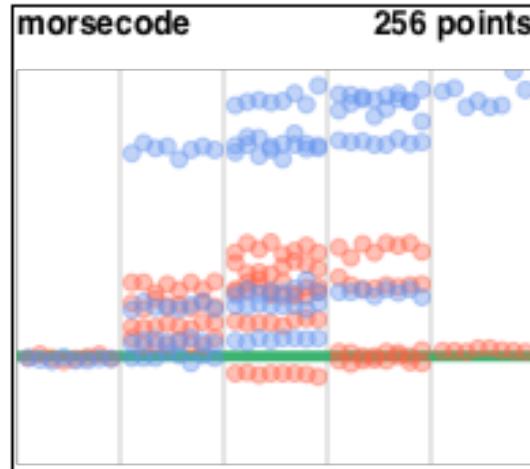
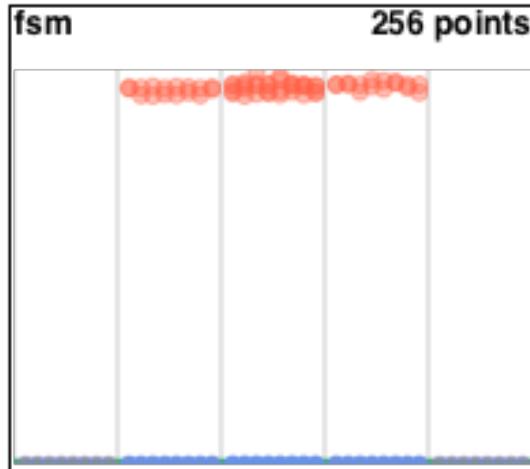


first-order (enforce type constructors)



erasure (ignore types)





KafKa: Gradual Typing for Objects



Benjamin Chung



Francesco Zappa Nardelli



Paley Li



Jan Vitek