Practice problems for the Mid-term

Disclaimer: You are allowed to reference any course material that you bring with you, but laptop usage is not allowed. Also, these problems are meant only to give a rough level of the problems on the mid-term.

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1. A point $x \in \mathbb{R}^n$ is said to ε-approximately lie in the convex hull of a given set of points $v_1, v_2, \ldots, v_m \in \mathbb{R}^n$, if there exists a perturbation of y of x, such that (a) every coordinate of y differs from the corresponding coordinate of x by at most ε (i.e., $|x_i - y_i| \leq \varepsilon$), and (b) y lies in the convex hull of $v_1, \ldots, v_n$.

   Show that there exists a polynomial (in m, n) time algorithm that can check if x ε-approximately lies in the convex hull of a given set of points.

2. Give an example of a matrix M and an initial vector $x^{(0)}$ for which the power method does not converge to the vector corresponding to the largest (in magnitude) eigenvalue. Also give an example in which the magnitudes of the eigenvalues are all distinct.

3. Consider the semidefinite programming relaxation for the max-cut problem we saw in class. When the graph is a triangle (3 vertices with all pairwise edges), show that the SDP has objective value $\geq 9/4 = 2.25$. Note that the true max cut value is 2.

4. What are all the eigenvalues of the adjacency matrix of (a) a complete graph on n vertices, (b) a complete bipartite graph with n vertices on each side?