Practice Final

Disclaimer: You are allowed to reference any course material/online sources. These problems are meant only to give a rough level of the problems on the final.

---

1. Let G be a complete bipartite graph with $\sqrt{n}$ vertices on one side (called U) and n vertices on the other (called V). Consider the lazy random walk with laziness parameter (probability of staying put) α on G, and let $\pi_\alpha(x)$ denote the stationary probability of a vertex x.

   (a) Consider some $u \in U$ and $v \in V$. What is $\pi_{1/2}(u)/\pi_{1/2}(v)$?

   (b) Is $\pi_{1/4}(u) = \pi_{1/2}(u)$? (In other words, does a walk that is lazier have a different stationary distribution?)

2. Consider a zero sum game between row and column players, in which the payoff matrix for the column player is given by
$$\begin{bmatrix} 1 & -2 & 5 \\ 2 & 1 & 3 \\ 0 & 4 & 2 \end{bmatrix}.$$
   The (i, j)th entry denotes the payoff to the column player if the row player plays strategy i and the column player plays j. What is the best strategy for the row player? (Note that his goal is to make sure the column player gets as little payoff as possible, as this is a zero-sum game.)

3. Consider the set $S \subset \mathbb{R}^n$, defined by
$$|x_1| + |x_2| + \cdots + |x_n| \leq 1,$$
   where $|\cdot|$ denotes the absolute value.

   (a) Show that S is convex by giving a set of linear inequalities that define S.

   (b) Describe a separation oracle for this set of inequalities