

$ \begin{aligned} \text{mod} &::= \mathbf{MODULE}(mname, \text{dep } \dots, \text{defn } \dots) \\ M &::= \text{mod } \dots \\ mname &::= \text{name} \\ \text{dep} &::= mname@ph \\ \text{deps} &::= \text{dep } \dots \\ \text{defn} &::= \langle dname@ph, \text{kind}, \text{ast} \rangle \\ \text{defns} &::= \text{defn } \dots \\ dname &::= \text{name} \\ \text{kind} &::= \mathbf{VALUE} \mid \mathbf{MACRO} \\ ph &::= \text{integer} \\ \\ \text{binding} &::= \langle ph, mname, dname \rangle \\ \text{bindings} &::= \text{binding } \dots \\ \\ \Sigma &::= \{ \text{cell } \dots \} \\ \text{cell} &::= \langle \text{binding}, \text{kind}, \text{val} \rangle \\ &\quad \mid \langle mname@ph \rangle \end{aligned} $	$ \begin{aligned} s\text{-exp} &::= \text{name} \mid (s\text{-exp } \dots) \\ \text{name} &::= \text{a token such as } \mathbf{x}, \mathbf{clock}, \text{ or } \mathbf{lambda} \\ \\ \text{ast} &::= \text{val} \\ &\quad \mid \mathbf{REF}(\text{binding}) \\ &\quad \mid \text{var} \\ &\quad \mid \mathbf{APP}(\text{ast}, \text{ast}, \dots) \\ \text{val} &::= \text{func} \\ &\quad \mid \mathbf{QUOTE}(\text{name}) \\ &\quad \mid \mathbf{SYNTAX}(stx) \\ &\quad \mid \mathbf{UNBOUND} \\ \text{func} &::= \mathbf{LAMBDA}(\text{var } \dots, \text{ast}) \\ \text{var} &::= \mathbf{VAR}(\text{name}) \\ \\ \text{stx} &::= \text{id} \mid (stx \dots) \\ \text{id} &::= \mathbf{STX}(\text{name}, \text{bindings}) \\ \\ \text{body} &::= stx@ph \end{aligned} $
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$\text{cmodule} : s\text{-exp } M \rightarrow M$

$\text{cmodule}[(\mathbf{module } mname \text{ } mname_{init} \text{ } s\text{-exp } \dots), M]$
 $= \text{cbody}[body \dots, mname, mname_{init}@0, \varepsilon, M, \Sigma_{init}]$
subject to $\text{visit}[mname_{init}, 0, M, \emptyset] = \Sigma_{init}$,
 $\text{import}[mname_{init}, 0, M] = \text{bindings}$,
 $\text{wrap}[s\text{-exp}, \text{bindings}]@0 \dots = body \dots$

$\text{cbody} : body \dots mname \text{ dep } \dots \text{ defn } \dots M \Sigma \rightarrow M$

$\text{cbody}[\varepsilon, mname, \text{deps}, \text{defns}, M, \Sigma] = M[mname \leftarrow \mathbf{MODULE}(mname, \text{deps}, \text{defns})]$

$\text{cbody}[(id_{beg} \text{ } stx \dots) @ph \text{ } body_{rest} \dots, mname, \text{deps}, \text{defns}, M, \Sigma]$
 $= \text{cbody}[stx@ph \dots \text{ } body_{rest} \dots, mname, \text{deps}, \text{defns}, M, \Sigma]$
subject to $\text{resolve}[id_{beg}, ph] = \langle ph, \mathbf{base}, \mathbf{begin} \rangle$

$\text{cbody}[(id_{bfs} \text{ } stx \dots) @ph \text{ } body_{rest} \dots, mname, \text{deps}, \text{defns}, M, \Sigma]$
 $= \text{cbody}[stx@ph+1 \dots \text{ } body_{rest} \dots, mname, \text{deps}, \text{defns}, M, \Sigma]$
subject to $\text{resolve}[id_{bfs}, ph] = \langle ph, \mathbf{base}, \mathbf{begin-for-syntax} \rangle$

$\text{cbody}[(id_{def} \text{ } id_{new} \text{ } stx) @ph \text{ } body_{rest} \dots, mname, \text{deps}, \text{defn } \dots, M, \Sigma]$
 $= \text{cbody}[body_{new} \dots, mname, \text{deps}, \text{defn } \dots \text{ } \text{defn}_{new}, M, \Sigma_{new}]$
subject to $\text{resolve}[id_{def}, ph] = \langle ph, \mathbf{base}, \mathbf{define} \rangle$,
 $\text{strip}[id_{new}] = dname$, $\text{cexpr}[stx, ph, \varepsilon, \Sigma] = \text{ast}$,
 $\langle dname@ph, \mathbf{VALUE}, \text{ast} \rangle = \text{defn}_{new}$,
 $\langle ph, mname, dname \rangle = \text{binding}_{new}$,
 $\text{add}[body_{rest}, \text{binding}_{new}] \dots = body_{new} \dots$,
 $\text{maybe-eval}[ph, \text{ast}, \Sigma] = \text{val}$,
 $\Sigma[\text{binding}_{new} \leftarrow \langle \mathbf{VALUE}, \text{val} \rangle] = \Sigma_{new}$

$\text{cbody}[(id_{\text{defstx}} id_{\text{new}} stx) @ph \text{body}_{\text{rest}} \dots, mname, \text{deps}, \text{defn} \dots, M, \Sigma]$
 $= \text{cbody}[\text{body}_{\text{new}} \dots, mname, \text{deps}, \text{defn} \dots \text{defn}_{\text{new}}, M, \Sigma_{\text{new}}]$
 subject to $\text{resolve}[(id_{\text{defstx}}, ph) = \langle ph, \text{base}, \text{define-syntax} \rangle,$
 $\text{strip}[id_{\text{new}}] = dname, \text{cexpr}[stx, ph+1, \varepsilon, \Sigma] = ast,$
 $\langle dname @ ph, \text{MACRO}, ast \rangle = \text{defn}_{\text{new}},$
 $\langle ph, mname, dname \rangle = \text{binding}_{\text{new}},$
 $\text{add}[\text{body}_{\text{rest}}, \text{binding}_{\text{new}}] \dots = \text{body}_{\text{new}} \dots,$
 $\text{eval}[ast, \Sigma] = val,$
 $\Sigma[\text{binding}_{\text{new}} \leftarrow \langle \text{MACRO}, val \rangle] = \Sigma_{\text{new}}$

$\text{cbody}[(id_{\text{macro}} stx \dots) @ph \text{body}_{\text{rest}} \dots, mname, \text{deps}, \text{defns}, M, \Sigma]$
 $= \text{cbody}[stx_{\text{new}} @ph \text{body}_{\text{rest}} \dots, mname, \text{deps}, \text{defns}, M, \Sigma]$
 subject to $\text{resolve}[id_{\text{macro}}, ph] = \text{binding},$
 $\Sigma(\text{binding}) = \langle \text{MACRO}, \text{func} \rangle,$
 $\text{eval}[\text{APP}(\text{func}, \text{SYNTAX}((id_{\text{macro}} stx \dots))), \Sigma] = \text{SYNTAX}(stx_{\text{new}})$

$\text{cbody}[(id_{\text{req}} stx_{\text{in}}) @ph \text{body}_{\text{rest}} \dots, mname, \text{dep} \dots, \text{defns}, M, \Sigma]$
 $= \text{cbody}[\text{body}_{\text{new}} \dots, mname, \text{dep} \dots \text{dep}_{\text{new}}, \text{defns}, M, \Sigma_{\text{new}}]$
 subject to $\text{resolve}[id_{\text{req}}, ph] = \langle ph, \text{base}, \text{require} \rangle,$
 $\text{parse-req}[stx_{\text{in}}, ph] = \langle mname_{\text{in}}, ph_{\Delta} \rangle,$
 $\text{visit}[mname_{\text{in}}, ph_{\Delta}, M, \Sigma] = \Sigma_{\text{new}},$
 $\text{import}[mname_{\text{in}}, ph_{\Delta}, M] = \text{bindings}_{\text{new}},$
 $\text{add}[\text{body}_{\text{rest}}, \text{bindings}_{\text{new}}] \dots = \text{body}_{\text{new}} \dots,$
 $mname_{\text{in}} @ ph_{\Delta} = \text{dep}_{\text{new}}$

$\text{cbody}[(id_{\text{mod}} id_{\text{sub}} id_{\text{init}} stx \dots) @ph \text{body}_{\text{rest}} \dots, mname, \text{deps}, \text{defns}, M, \Sigma]$
 $= \text{cbody}[\text{body}_{\text{rest}} \dots, mname, \text{deps}, \text{defns}, M_{\text{new}}, \Sigma]$
 subject to $\text{resolve}[id_{\text{mod}}, ph] = \langle ph, \text{base}, \text{module} \rangle,$
 $\text{strip}[id_{\text{sub}}] = mname_{\text{sub}}, \text{strip}[id_{\text{init}}] = mname_{\text{init}},$
 $\text{cmodule}[(\text{module } mname_{\text{sub}} mname_{\text{init}} \text{strip}[stx] \dots), M] = M_{\text{new}}$

$\text{cbody}[(id_{\text{mod}} id_{\text{sub}} id_{\text{init}} stx \dots) @ph \text{body}_{\text{rest}} \dots, mname, \text{deps}, \text{defns}, M, \Sigma]$
 $= \text{cmodule}[(\text{module } mname_{\text{sub}} mname_{\text{init}} \text{strip}[stx] \dots), M_{\text{new}}]$
 subject to $\text{resolve}[id_{\text{mod}}, ph] = \langle ph, \text{base}, \text{module}^* \rangle,$
 $\text{strip}[id_{\text{sub}}] = mname_{\text{sub}}, \text{strip}[id_{\text{init}}] = mname_{\text{init}},$
 $\text{cbody}[\text{body}_{\text{rest}} \dots, mname, \text{deps}, \text{defns}, M, \Sigma] = M_{\text{new}}$

$\text{cbody}[(id_{\text{mod}} id_{\text{sub}} () stx \dots) @ph \text{body}_{\text{rest}} \dots, mname, \text{deps}, \text{defns}, M, \Sigma]$
 $= \text{cbody}[\text{shift}[stx, 0-ph] @ 0 \dots, mname_{\text{sub}}, mname @ 0-ph, \varepsilon, M_{\text{new}}, \Sigma_{\text{init}}]$
 subject to $\text{resolve}[id_{\text{mod}}, ph] = \langle ph, \text{base}, \text{module}^* \rangle,$
 $\text{strip}[id_{\text{sub}}] = mname_{\text{sub}},$
 $\text{cbody}[\text{body}_{\text{rest}} \dots, mname, \text{deps}, \text{defns}, M, \Sigma] = M_{\text{new}},$
 $\text{visit}[mname, 0-ph, M_{\text{new}}, \emptyset] = \Sigma_{\text{init}}$

$\text{cexpr} : stx \ ph \ env \ \Sigma \rightarrow ast$

$\text{cexpr}[id, ph, env, \Sigma] = var$
 subject to $\text{env-lookup}[env, \text{VAR}(\text{strip}[id])] = var$

$\text{cexpr}[[id, ph, env, \Sigma] = \mathbf{REF}(binding)$
 subject to $\text{resolve}[[id, ph] = binding$

$\text{cexpr}[[id_{lambda} (id \dots) stx], ph, name_{env} \dots, \Sigma]$
 $= \mathbf{LAMBDA}(\mathbf{VAR}(\text{strip}[[id]) \dots, \text{cexpr}[[stx, ph, \mathbf{VAR}(\text{strip}[[id]) \dots name_{env} \dots, \Sigma]])$
 subject to $\text{resolve}[[id_{lambda}, ph] = \langle ph, \mathbf{base}, \mathbf{lambda} \rangle$

$\text{cexpr}[[id_{quote} stx], ph, env, \Sigma] = \mathbf{QUOTE}(\text{strip}[[stx]])$
 subject to $\text{resolve}[[id_{quote}, ph] = \langle ph, \mathbf{base}, \mathbf{quote} \rangle$

$\text{cexpr}[[id_{stx} stx], ph, env, \Sigma] = \mathbf{SYNTAX}(stx)$
 subject to $\text{resolve}[[id_{stx}, ph] = \langle ph, \mathbf{base}, \mathbf{syntax} \rangle$

$\text{cexpr}[[id_{macro} stx \dots], ph, env, \Sigma] = \text{cexpr}[[stx_{new}, ph, env, \Sigma]$
 subject to $\text{resolve}[[id_{macro}, ph] = binding,$
 $\Sigma(binding) = \langle \mathbf{MACRO}, func \rangle,$
 $\text{eval}[[\mathbf{APP}(func, \mathbf{SYNTAX}((id_{macro} stx \dots))], \Sigma] = \mathbf{SYNTAX}(stx_{new})$

$\text{cexpr}[[stx \dots], ph, env, \Sigma] = \mathbf{APP}(\text{cexpr}[[stx, ph, env, \Sigma], \dots)$

$\text{cexpr}[[id, ph, env, \Sigma] = \mathbf{UNBOUND}$

$\text{eval} : ast \ \Sigma \rightarrow val$

$\text{eval}[[val, \Sigma] = val$

$\text{eval}[[\mathbf{REF}(binding), \Sigma] = val$

subject to $\Sigma(binding) = \langle \mathbf{VALUE}, val \rangle$

$\text{eval}[[\mathbf{APP}(ast, ast_{arg}, \dots), \Sigma] = \text{eval}[[ast_{body} [var \dots \leftarrow val \dots], \Sigma]$

subject to $\text{eval}[[ast, \Sigma] = \mathbf{LAMBDA}(var \dots, ast_{body}),$

$\text{eval}[[ast_{arg}, \Sigma] \dots = val \dots$

$\text{maybe-eval} : ph \ ast \ \Sigma \rightarrow val$

$\text{maybe-eval}[[0, ast, \Sigma] = \mathbf{UNBOUND}$

$\text{maybe-eval}[[ph, ast, \Sigma] = \text{eval}[[ast, \Sigma]$

$\text{ast-shift} : ast \ ph \rightarrow ast$

$\text{ast-shift}[[var, ph] = var$

$\text{ast-shift}[[\mathbf{LAMBDA}(var \dots, ast), ph] = \mathbf{LAMBDA}(var \dots, \text{ast-shift}[[ast, ph]])$

$\text{ast-shift}[[\mathbf{QUOTE}(name), ph] = \mathbf{QUOTE}(name)$

$\text{ast-shift}[\mathbf{APP}(ast, \dots), ph] = \mathbf{APP}(\text{ast-shift}[ast, ph], \dots)$

$\text{ast-shift}[\mathbf{REF}(\langle ph_{ref}, mname, dname \rangle), ph] = \mathbf{REF}(\langle ph_{ref}+ph, mname, dname \rangle)$

$\text{ast-shift}[\mathbf{SYNTAX}(stx), ph] = \mathbf{SYNTAX}(\text{shift}[stx, ph])$

$\text{ast-shift}[\mathbf{UNBOUND}, ph] = \mathbf{UNBOUND}$

$\text{visit} : mname\ ph\ M\ \Sigma \rightarrow \Sigma$

$\text{visit}[mname, ph_{\Delta}, M, \Sigma] = \Sigma$
subject to $\Sigma(mname@ph_{\Delta}) = \mathbf{READY}$

$\text{visit}[mname, ph_{\Delta}, M, \Sigma] = \text{visit-body}[defn\ \dots, mname, ph_{\Delta}, \Sigma_{deps}]$
subject to $M(mname) = \mathbf{MODULE}(mname, deps, defn\ \dots),$
 $\Sigma[mname@ph_{\Delta} \leftarrow \mathbf{READY}] = \Sigma_{init},$
 $\text{visit}^*[deps, ph_{\Delta}, M, \Sigma_{init}] = \Sigma_{deps}$

$\text{visit}^* : deps\ ph\ M\ \Sigma \rightarrow \Sigma$

$\text{visit}^*[\varepsilon, ph_{\Delta}, M, \Sigma] = \Sigma$

$\text{visit}^*[mname@ph\ dep\ \dots, ph_{\Delta}, M, \Sigma] = \text{visit}^*[dep\ \dots, ph_{\Delta}, M, \Sigma_{new}]$
subject to $\text{visit}[mname, ph_{\Delta}+ph, M, \Sigma] = \Sigma_{new}$

$\text{visit-body} : defns\ name\ ph\ \Sigma \rightarrow \Sigma$

$\text{visit-body}[\varepsilon, mname, ph_{\Delta}, \Sigma] = \Sigma$

$\text{visit-body}[\langle dname@ph_{def}, \mathbf{VALUE}, ast \rangle defn\ \dots, mname, ph_{\Delta}, \Sigma]$
 $= \text{visit-body}[defn\ \dots, mname, ph_{\Delta}, \Sigma]$
subject to $ph_{\Delta}+ph_{def} = 0$

$\text{visit-body}[\langle dname@ph_{def}, kind, ast \rangle defn\ \dots, mname, ph_{\Delta}, \Sigma]$
 $= \text{visit-body}[defn\ \dots, mname, ph_{\Delta}, \Sigma]$
subject to $\text{pmax}[-1, ph_{\Delta}+ph_{def}] = -1$

$\text{visit-body}[\langle dname@ph_{def}, kind, ast \rangle defn\ \dots, mname, ph_{\Delta}, \Sigma]$
 $= \text{visit-body}[defn\ \dots, mname, ph_{\Delta}, \Sigma_{new}]$
subject to $ph_{\Delta}+ph_{def} = ph, \text{eval}[\text{shift}[ast, ph_{\Delta}], \Sigma] = val,$
 $\Sigma[\langle ph, mname, dname \rangle \leftarrow \langle kind, val \rangle] = \Sigma_{new}$

$\text{run} : mname\ ph\ M\ \Sigma \rightarrow \Sigma$

$\text{run}[mname, ph_{\Delta}, M, \Sigma] = \Sigma$
subject to $\Sigma(mname@ph_{\Delta}) = \mathbf{READY}$

$$\begin{aligned} \text{run}[mname, ph_{\Delta}, M, \Sigma] &= \text{run-body}[defn \dots, mname, ph_{\Delta}, \Sigma_{deps}] \\ \text{subject to } M(mname) &= \mathbf{MODULE}(mname, deps, defn \dots), \\ \Sigma[mname@ph_{\Delta} \leftarrow \mathbf{READY}] &= \Sigma_{init}, \\ \text{run}^*[deps, ph_{\Delta}, M, \Sigma_{init}] &= \Sigma_{deps} \end{aligned}$$

$$\text{run}^* : deps \ ph \ M \ \Sigma \rightarrow \Sigma$$

$$\text{run}^*[\epsilon, ph_{\Delta}, M, \Sigma] = \Sigma$$

$$\begin{aligned} \text{run}^*[mname@ph \ dep \dots, ph_{\Delta}, M, \Sigma] &= \text{run}^*[dep \dots, ph_{\Delta}, M, \Sigma_{new}] \\ \text{subject to } \text{run}[mname, ph_{\Delta}+ph, M, \Sigma] &= \Sigma_{new} \end{aligned}$$

$$\text{run-body} : defns \ name \ ph \ \Sigma \rightarrow \Sigma$$

$$\text{run-body}[\epsilon, mname, ph_{\Delta}, \Sigma] = \Sigma$$

$$\begin{aligned} \text{run-body}[\langle dname@ph_{def}, \mathbf{VALUE}, ast \rangle defn \dots, mname, ph_{\Delta}, \Sigma] \\ = \text{run-body}[defn \dots, mname, ph_{\Delta}, \Sigma_{new}] \\ \text{subject to } ph_{\Delta}+ph_{def} = 0, \text{eval}[\text{shift}[ast, ph_{\Delta}], \Sigma] = val, \\ \Sigma[\langle 0, mname, dname \rangle \leftarrow \langle \mathbf{VALUE}, val \rangle] = \Sigma_{new} \end{aligned}$$

$$\begin{aligned} \text{run-body}[defn_{skip} defn \dots, mname, ph_{\Delta}, \Sigma] \\ = \text{run-body}[defn \dots, mname, ph_{\Delta}, \Sigma] \end{aligned}$$

$$\text{wrap} : s\text{-exp} \ bindings \rightarrow stx$$

$$\text{wrap}[name, bindings] = \mathbf{STX}(name, bindings)$$

$$\text{wrap}[(s\text{-exp} \dots), bindings] = (\text{wrap}[s\text{-exp}, bindings] \dots)$$

$$\text{strip} : stx \rightarrow s\text{-exp}$$

$$\text{strip}[\mathbf{STX}(name, bindings)] = name$$

$$\text{strip}[(stx \dots)] = (\text{strip}[stx] \dots)$$

$$\text{add-to-stx} : stx \ bindings \rightarrow stx$$

$$\begin{aligned} \text{add-to-stx}[\mathbf{STX}(name, binding \dots), binding_{new} \dots] \\ = \mathbf{STX}(name, binding \dots binding_{new} \dots) \end{aligned}$$

$$\text{add-to-stx}[(stx \dots), bindings] = (\text{add-to-stx}[stx, bindings] \dots)$$

$$\text{add} : body \ bindings \rightarrow body$$

$$\text{add}[stx@ph, bindings] = \text{add-to-stx}[stx, bindings]@ph$$

resolve : $id\ ph \rightarrow maybe\text{-}binding$

resolve[**STX**($dname, binding_{pre} \dots binding\ binding_{post} \dots$), ph] = $binding$
subject to $binding = \langle ph, mname, dname \rangle$

resolve[id, ph] = #f

shift : $stx\text{-or-}ast\ ph \rightarrow stx\text{-or-}ast$

shift[**STX**($name, \langle ph, mname, dname \rangle \dots$), ph_{shift}]
= **STX**($name, \langle ph+ph_{shift}, mname, dname \rangle \dots$)

shift[($stx \dots$), ph_{shift}] = (shift[stx, ph_{shift}] ...)

shift[ast, ph_{shift}] = ast-shift[ast, ph_{shift}]

parse-req : $stx\ ph \rightarrow \langle name, ph \rangle$

parse-req[id_{in}, ph] = $\langle strip[id_{in}], ph \rangle$

parse-req[($id_{fs}\ stx_{in}$), ph] = parse-req[$stx_{in}, ph+1$]
subject to $strip[id_{fs}] = \text{for-syntax}$

parse-req[($id_{ft}\ stx_{in}$), ph] = parse-req[$stx_{in}, ph-1$]
subject to $strip[id_{ft}] = \text{for-template}$

import : $mname\ ph_{\Delta}\ M \rightarrow bindings$

import[$mname, ph_{\Delta}, M$] = $\langle ph_{\Delta}+ph_{def}, mname, dname \rangle \dots$
subject to $M(mname) = \text{MODULE}(mname, dep \dots, defn \dots)$,
 $defn \dots = \langle dname@ph_{def}, kind, ast \rangle \dots$