

Random Numbers

Our goal is to turn a plaintext into a ciphertext that looks *random*
... but isn't really random

That's also what a **pseudo-random number generator (PRNG)** does

PRNGs

```
> random.seed(74658319934)
> random(256)
243
> random(256)
122
> random(256)
187
> random(256)
43
> random(256)
200
> random(256)
229
```

PRNGs

```
> random.seed(74658319934)
> random(256)
243
> random(256)
122
> random(256)
187
> random(256)
43
> random(256)
200
> random(256)
229
```

Using a PRNG sequence to
to encrypt is called a
stream cipher

Stream Cipher

Let $K = K_0, K_1, \dots, K_i, \dots$ be a sequence of random numbers
that is generated from a seed 

Convert the i th plaintext  element to the i th ciphertext  element:

$$CT_i = PT_i \oplus K_i$$

Stream Cipher

Let $K = K_0, K_1, \dots, K_i, \dots$ be a sequence of random numbers
that is generated from a seed 

Convert the i th plaintext  element to the i th ciphertext  element:

$$\text{CT}_i = \text{PT}_i \oplus K_i$$

$$\begin{array}{rcl} 0 & \oplus & 0 & = & 0 \\ 1 & \oplus & 0 & = & 1 \\ 0 & \oplus & 1 & = & 1 \\ 1 & \oplus & 1 & = & 0 \end{array}$$

Stream Cipher

Let $K = K_0, K_1, \dots, K_i, \dots$ be a sequence of random numbers
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Convert the i th plaintext  element to the i th ciphertext  element:

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$$\begin{array}{rcl} 0 & \oplus & 0 = 0 \\ 0 & \oplus & 1 = 1 \\ 1 & \oplus & 0 = 1 \\ 1 & \oplus & 1 = 0 \end{array}$$

Stream Cipher

Let $K = K_0, K_1, \dots, K_i, \dots$ be a sequence of random numbers
that is generated from a seed 

Convert the i th plaintext  element to the i th ciphertext  element:

$$CT_i = PT_i \oplus K_i$$

$$\begin{array}{rcl} 0 & = & 0 \oplus 0 \\ 0 & = & 1 \oplus 1 \\ 1 & = & 0 \oplus 1 \\ 1 & = & 1 \oplus 0 \end{array}$$

Stream Cipher

Let $K = K_0, K_1, \dots, K_i, \dots$ be a sequence of random numbers
that is generated from a seed 

Convert the i th plaintext  element to the i th ciphertext  element:

$$CT_i = PT_i \oplus K_i$$

$$\begin{array}{rcl} 0 & \oplus & 0 & = & 0 \\ 0 & \oplus & 1 & = & 1 \\ 1 & \oplus & 0 & = & 1 \\ 1 & \oplus & 1 & = & 0 \end{array}$$

Stream Cipher

Let $K = K_0, K_1, \dots, K_i, \dots$ be a sequence of random numbers
that is generated from a seed 

Convert the i th plaintext  element to the i th ciphertext  element:

$$CT_i = PT_i \oplus K_i$$

$$\begin{array}{rcl} 0 & = & 0 \oplus 0 \\ 0 & = & 1 \oplus 1 \\ 1 \oplus 0 & = & 1 \\ 1 \oplus 1 & = & 0 \end{array}$$

Stream Cipher

Let $K = K_0, K_1, \dots, K_i, \dots$ be a sequence of random numbers
that is generated from a seed 

Convert the i th plaintext  element to the i th ciphertext  element:

$$CT_i = PT_i \oplus K_i$$

$$\begin{array}{r} 12 \\ \oplus \\ 9 \end{array}$$

Stream Cipher

Let $K = K_0, K_1, \dots, K_i, \dots$ be a sequence of random numbers
that is generated from a seed 

Convert the i th plaintext  element to the i th ciphertext  element:

$$CT_i = PT_i \oplus K_i$$

$$\begin{array}{r} 1100 \\ \oplus \\ 1001 \end{array}$$

Stream Cipher

Let $K = K_0, K_1, \dots, K_i, \dots$ be a sequence of random numbers
that is generated from a seed 

Convert the i th plaintext  element to the i th ciphertext  element:

$$CT_i = PT_i \oplus K_i$$

$$\begin{array}{r} 1100 \\ \oplus\oplus\oplus\oplus \\ 1001 \end{array}$$

Stream Cipher

Let $K = K_0, K_1, \dots, K_i, \dots$ be a sequence of random numbers
that is generated from a seed 

Convert the i th plaintext  element to the i th ciphertext  element:

$$CT_i = PT_i \oplus K_i$$

$$\begin{array}{r} 1100 \\ \oplus\oplus\oplus\oplus \\ 1001 \\ \hline 0101 \end{array}$$

Stream Cipher

Let $K = K_0, K_1, \dots, K_i, \dots$ be a sequence of random numbers
that is generated from a seed 

Convert the i th plaintext  element to the i th ciphertext  element:

$$CT_i = PT_i \oplus K_i$$

$$\begin{array}{r} 12 \\ \oplus \\ 9 \\ \hline 5 \end{array}$$

Stream Cipher

Let $K = K_0, K_1, \dots, K_i, \dots$ be a sequence of random numbers
that is generated from a seed 

Convert the i th plaintext \boxed{PT} element to the i th ciphertext \boxed{CT} element:

$$\boxed{CT_i} = \boxed{PT_i} \oplus \boxed{K_i}$$

Since \oplus is its own inverse:

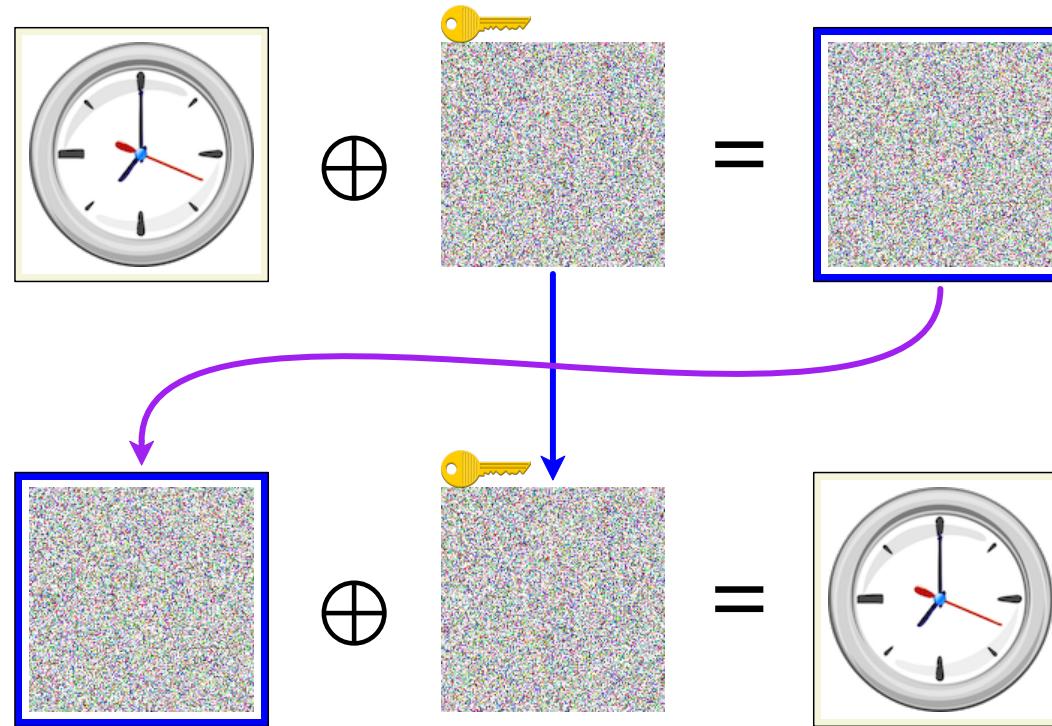
$$\boxed{PT_i} = \boxed{CT_i} \oplus \boxed{K_i}$$

Treating \oplus as mapping over sequences

$$\boxed{CT} = \boxed{PT} \oplus \boxed{K}$$

$$\boxed{PT} = \boxed{CT} \oplus \boxed{K}$$

Stream Cipher on an Image



Well, That Was Easy!

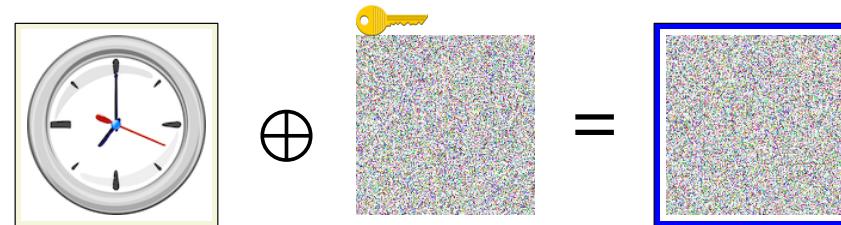
Why aren't we done?

Problem #1: it turns out that \oplus uses up a key
unless the PRNG has an extra feature making it more general

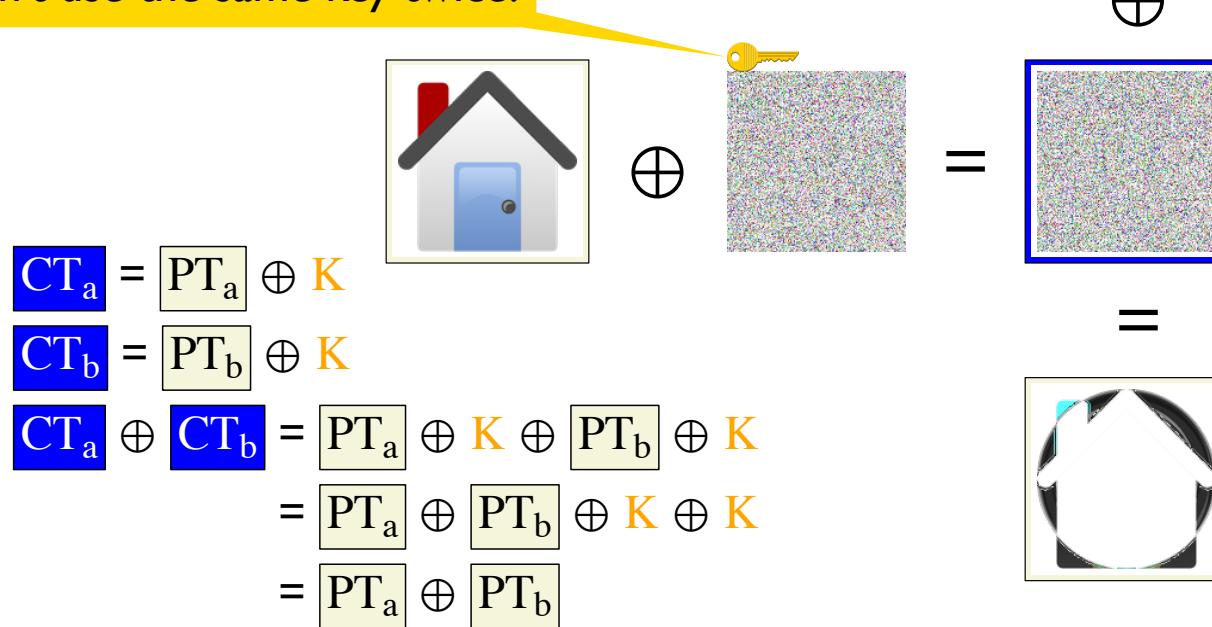
Problem #2: we're getting confidentiality, but not integrity
so, need to combine with something else, not today

Problem #3: we need a really good PRNG
a general building block, so we'll look more today

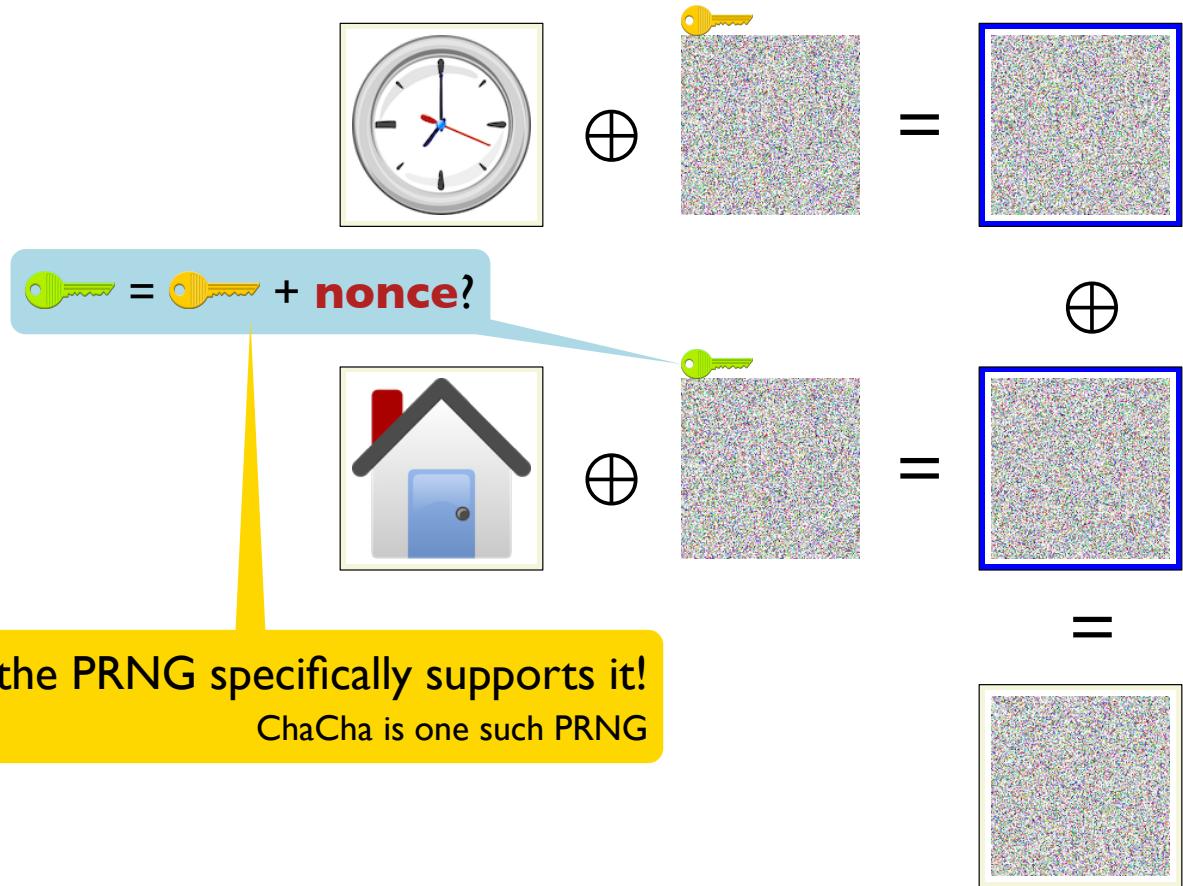
Misuse of Stream Cipher and Key



Don't use the same key twice!



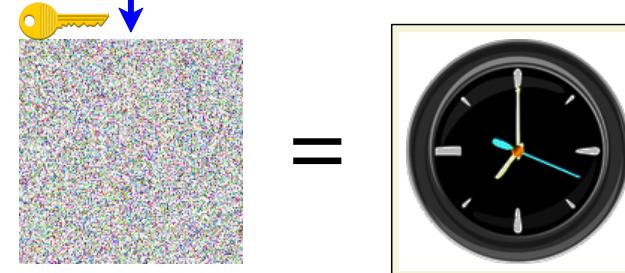
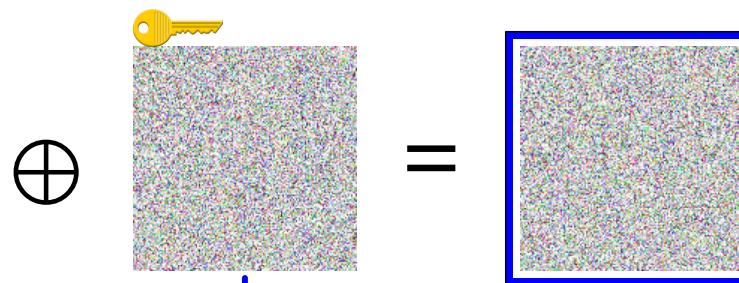
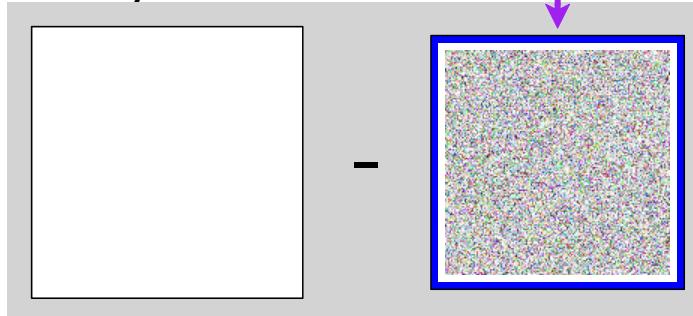
Potentially Correct Use of Stream Cipher and Key



Stream Cipher Lack of Integrity



Mallory



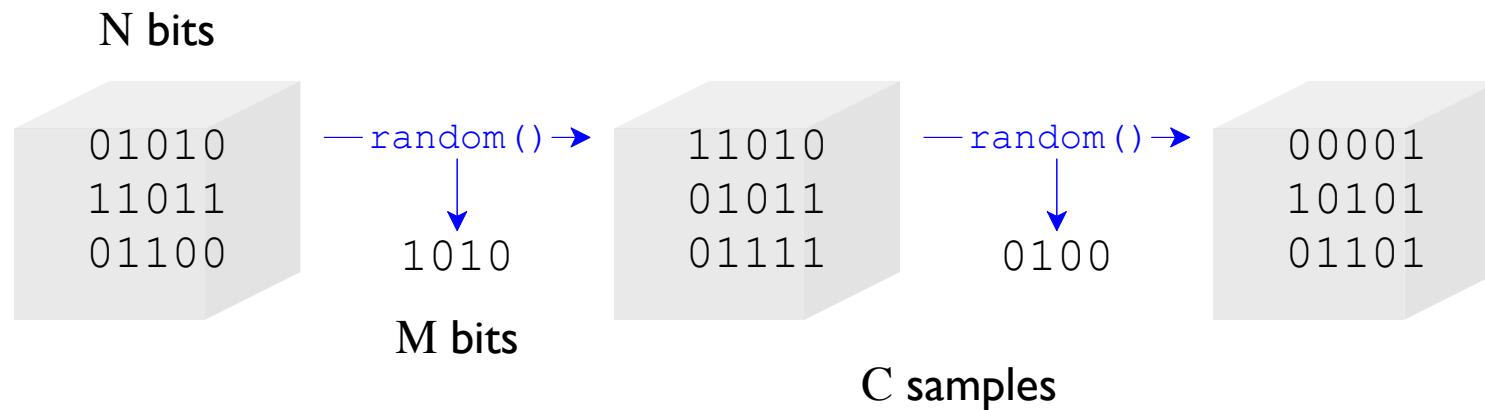
Again, we'll need more ingredients...

Creating a Really Good PRNG

We need a **cryptographically secure PRNG (CSPRNG)**

- Cannot predict next output from previous outputs
- Passes statistical randomness tests
- Cannot use current state to predict *previous* output

PRNG

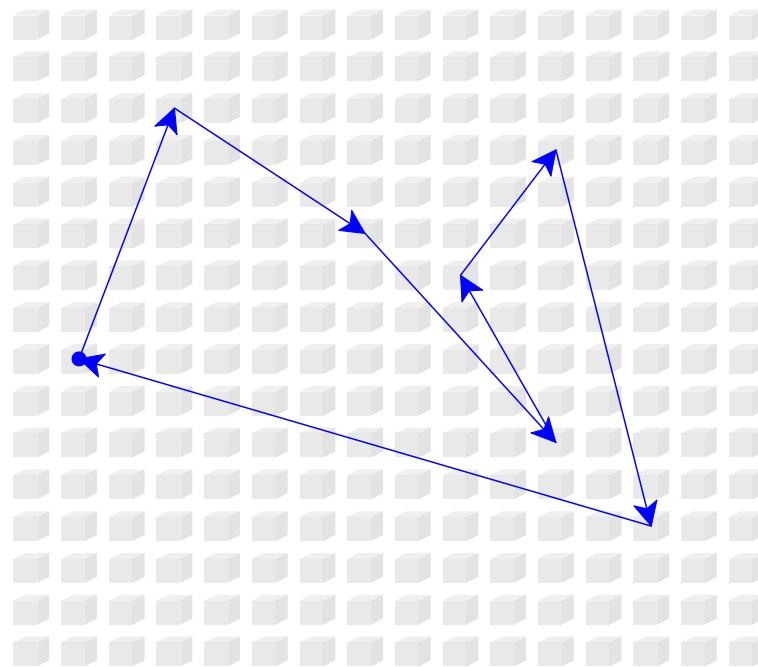


$2^{C \times M}$ possible sequences

2^N possible states

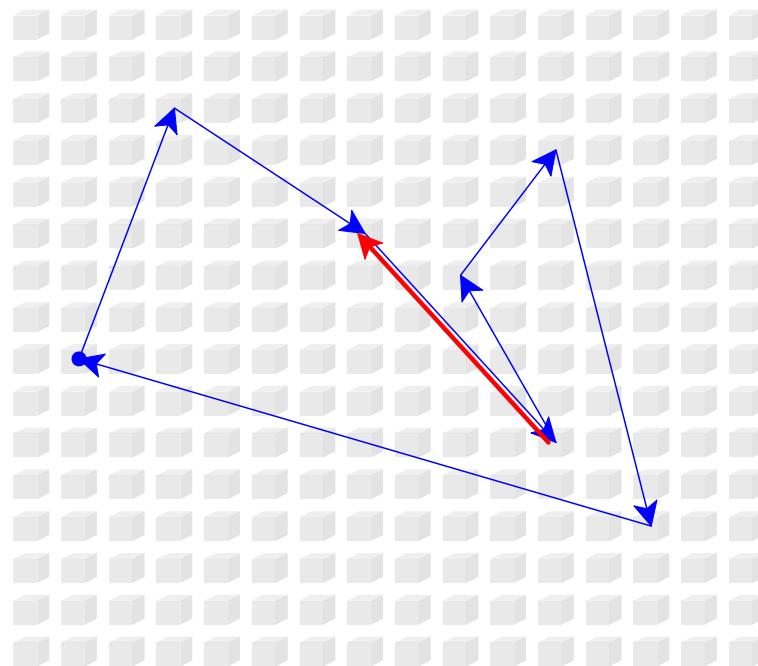
⇒ lots of overlap when $C \times M \gg N$

PRNG States



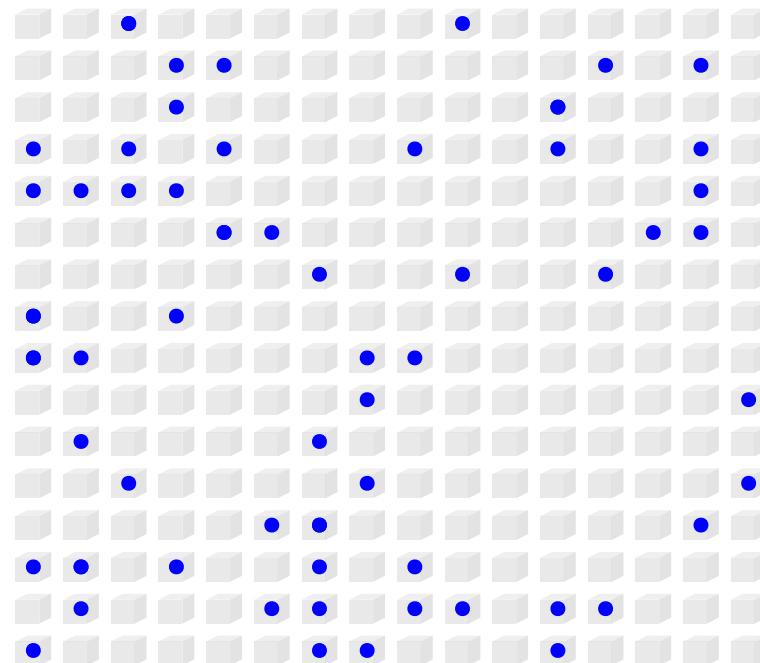
Cycle is inevitable, but
we want it to be as long
as possible

PRNG States



CSPRNG implies that
you can't reverse an arrow
even if you see the state

PRNG States



Key to initialize usually
 $< N$ bits, so only
some starts are possible

Historical PRNG: RC4

	i = 0 j = 0							
0	1	2	3	4	5	6	7	
8	9	10	11	12	13	14	15	
16	17	18	19	20	21	22	23	
24	25	26	27	28	29	30	31	
32	33	34	35	36	37	38	39	
40	41	42	43	44	45	46	47	
48	49	50	51	52	53	54	55	
56	57	58	59	60	61	62	63	
64	65	66	67	68	69	70	71	
72	73	74	75	76	77	78	79	
80	81	82	83	84	85	86	87	
88	89	90	91	92	93	94	95	
96	97	98	99	100	101	102	103	
104	105	106	107	108	109	110	111	
112	113	114	115	116	117	118	119	
120	121	122	123	124	125	126	127	
128	129	130	131	132	133	134	135	
136	137	138	139	140	141	142	143	
144	145	146	147	148	149	150	151	
152	153	154	155	156	157	158	159	
160	161	162	163	164	165	166	167	
168	169	170	171	172	173	174	175	
176	177	178	179	180	181	182	183	
184	185	186	187	188	189	190	191	
192	193	194	195	196	197	198	199	
200	201	202	203	204	205	206	207	
208	209	210	211	212	213	214	215	
216	217	218	219	220	221	222	223	
224	225	226	227	228	229	230	231	
232	233	234	235	236	237	238	239	
240	241	242	243	244	245	246	247	
248	249	250	251	252	253	254	255	

RC4 is “Ron’s Code”

Used in the 1990s, but several vulnerabilities have been found, so don’t use it

Historical PRNG: RC4

i = 0 j = 0							
0	1	2	3	4	5	6	7
8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23
24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71
72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87
88	89	90	91	92	93	94	95
96	97	98	99	100	101	102	103
104	105	106	107	108	109	110	111
112	113	114	115	116	117	118	119
120	121	122	123	124	125	126	127
128	129	130	131	132	133	134	135
136	137	138	139	140	141	142	143
144	145	146	147	148	149	150	151
152	153	154	155	156	157	158	159
160	161	162	163	164	165	166	167
168	169	170	171	172	173	174	175
176	177	178	179	180	181	182	183
184	185	186	187	188	189	190	191
192	193	194	195	196	197	198	199
200	201	202	203	204	205	206	207
208	209	210	211	212	213	214	215
216	217	218	219	220	221	222	223
224	225	226	227	228	229	230	231
232	233	234	235	236	237	238	239
240	241	242	243	244	245	246	247
248	249	250	251	252	253	254	255

```
def S = Array(0..255)  
def i = 0  
def j = 0
```

Historical PRNG: RC4

i = 0 j = 0							
0	1	2	3	4	5	6	7
8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23
24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63
64	65	66	67	68	69	70	71
72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87
88	89	90	91	92	93	94	95
96	97	98	99	100	101	102	103
104	105	106	107	108	109	110	111
112	113	114	115	116	117	118	119
120	121	122	123	124	125	126	127
128	129	130	131	132	133	134	135
136	137	138	139	140	141	142	143
144	145	146	147	148	149	150	151
152	153	154	155	156	157	158	159
160	161	162	163	164	165	166	167
168	169	170	171	172	173	174	175
176	177	178	179	180	181	182	183
184	185	186	187	188	189	190	191
192	193	194	195	196	197	198	199
200	201	202	203	204	205	206	207
208	209	210	211	212	213	214	215
216	217	218	219	220	221	222	223
224	225	226	227	228	229	230	231
232	233	234	235	236	237	238	239
240	241	242	243	244	245	246	247
248	249	250	251	252	253	254	255

```
j := 0
for (i: 0..255):
    j := (j + S[i] + key[i % key.length]) % 256
    swap(S[i], S[j])

i := 0
j := 0
```

This and later code based on
<https://en.wikipedia.org/wiki/RC4>

Historical PRNG: RC4

		i = 0	j = 0
75		51	132
74		157	192
91		200	29
191		130	168
188		144	74
215		228	131
21		34	123
56		41	236
36		185	143
202		204	143
106		92	216
82		35	186
142		110	103
86		77	137
47		8	182
25		35	126
183		35	138
40		135	126
43		113	126
23		255	172
141		172	84
30		65	104
17		32	78
95		45	135
76		6	113
16		65	255
53		36	152
201		36	116
170		46	189
1		7	217
248		108	20
249		231	227
221		113	55
52		172	55
124		227	227
10		55	227
211		147	55
61		147	55
147		55	147

```
def key = "Secret"

j := 0
for (i: 0..255):
    j := (j + S[i] + key[i % key.length]) % 256
    swap(S[i], S[j])

i := 0
j := 0
```

Historical PRNG: RC4

		i = 0	j = 0
75		51	132
74		157	192
91		200	29
191		168	
188		74	243
215		131	131
21		228	228
56		18	18
36		112	112
202		130	130
106		144	144
82		91	143
142		34	236
86		41	34
47		185	41
25		204	185
183		92	204
40		191	92
43		216	216
23		186	186
141		14	14
30		110	110
17		77	77
95		8	8
76		35	35
16		35	35
53		248	248
201		129	129
170		68	68
1		48	48
248		254	254
249		164	164
221		250	250
52		167	167
125		107	107
177		99	99
166		147	147
232		124	124
120		10	10
211		211	211
61		61	61
249		147	147

any length, but between 5 and 16 bytes is common

```
def key = "Secret"

j := 0
for (i: 0..255):
    j := (j + S[i] + key[i % key.length]) % 256
    swap(S[i], S[j])

i := 0
j := 0
```

Historical PRNG: RC4

		i = 0	j = 0							
		75		51	132	157	192	200	29	168
		74		243	131	228	18	112	130	144
		91		143	236	34	41	185	204	92
		191		216	186	14	110	77	8	35
		188		27	103	137	182	64	59	105
		215		247	238	126	138	26	227	55
		21		84	104	78	135	113	255	172
		56		89	187	28	62	32	45	65
		36		251	152	116	189	7	108	46
		202		162	159	83	31	154	11	231
		106		13	0	217	20	229	102	118
		82		85	176	97	214	151	6	4
		142		245	134	60	225	165	3	39
		86		101	90	127	197	72	117	146
		47		195	42	128	100	253	174	209
		25		239	114	219	244	234	163	190
		183		235	54	98	153	121	123	38
		40		180	179	139	203	70	5	24
		43		199	224	213	210	220	173	241
		23		88	196	79	242	58	9	73
		141		160	193	181	19	233	63	80
		30		81	111	226	175	150	207	222
		17		119	230	96	71	87	133	198
		95		169	155	212	66	49	205	2
		76		115	37	194	57	22	223	178
		16		12	93	237	240	33	206	69
		53		158	148	15	122	136	161	246
		201		44	171	67	184	109	252	50
		170		145	149	140	94	218	156	208
		1		129	68	48	254	164	250	167
		248		125	177	166	232	120	107	99
		249		221	52	124	10	211	61	147

```
def next_byte():
    i := (i+1) mod 256
    j := (j+S[i]) mod 256
    swap(S[i], S[j])
    return S[(S[i] + S[j]) mod 256]
```

Historical PRNG: RC4

i = 1 j = 51											
75	78	132	157	192	200	29	168				
74	243	131	228	18	112	130	144				
91	143	236	34	41	185	204	92				
191	216	186	14	110	77	8	35				
188	27	103	137	182	64	59	105				
215	247	238	126	138	26	227	55				
21	84	104	51	135	113	255	172				
56	89	187	28	62	32	45	65				
36	251	152	116	189	7	108	46				
202	162	159	83	31	154	11	231				
106	13	0	217	20	229	102	118				
82	85	176	97	214	151	6	4				
142	245	134	60	225	165	3	39				
86	101	90	127	197	72	117	146				
47	195	42	128	100	253	174	209				
25	239	114	219	244	234	163	190				
183	235	54	98	153	121	123	38				
40	180	179	139	203	70	5	24				
43	199	224	213	210	220	173	241				
23	88	196	79	242	58	9	73				
141	160	193	181	19	233	63	80				
30	81	111	226	175	150	207	222				
17	119	230	96	71	87	133	198				
95	169	155	212	66	49	205	2				
76	115	37	194	57	22	223	178				
16	12	93	237	240	33	206	69				
53	158	148	15	122	136	161	246				
201	44	171	67	184	109	252	50				
170	145	149	140	94	218	156	208				
1	129	68	48	254	164	250	167				
248	125	177	166	232	120	107	99				
249	221	52	124	10	211	61	147				

```
def next_byte():
    i := (i+1) mod 256
    j := (j+S[i]) mod 256
    swap(S[i], S[j])
    return S[(S[i] + S[j]) mod 256]

next_byte() = 235
```

Historical PRNG: RC4

i = 2 j = 183									
75	78	198	157	192	200	29	168		
74	243	131	228	18	112	130	144		
91	143	236	34	41	185	204	92		
191	216	186	14	110	77	8	35		
188	27	103	137	182	64	59	105		
215	247	238	126	138	26	227	55		
21	84	104	51	135	113	255	172		
56	89	187	28	62	32	45	65		
36	251	152	116	189	7	108	46		
202	162	159	83	31	154	11	231		
106	13	0	217	20	229	102	118		
82	85	176	97	214	151	6	4		
142	245	134	60	225	165	3	39		
86	101	90	127	197	72	117	146		
47	195	42	128	100	253	174	209		
25	239	114	219	244	234	163	190		
183	235	54	98	153	121	123	38		
40	180	179	139	203	70	5	24		
43	199	224	213	210	220	173	241		
23	88	196	79	242	58	9	73		
141	160	193	181	19	233	63	80		
30	81	111	226	175	150	207	222		
17	119	230	96	71	87	133	132		
95	169	155	212	66	49	205	2		
76	115	37	194	57	22	223	178		
16	12	93	237	240	33	206	69		
53	158	148	15	122	136	161	246		
201	44	171	67	184	109	252	50		
170	145	149	140	94	218	156	208		
1	129	68	48	254	164	250	167		
248	125	177	166	232	120	107	99		
249	221	52	124	10	211	61	147		

```
def next_byte():
    i := (i+1) mod 256
    j := (j+S[i]) mod 256
    swap(S[i], S[j])
    return S[(S[i] + S[j]) mod 256]

next_byte() = 159
```

Historical PRNG: RC4

i = 3 j = 84											
75	78	198	20	192	200	29	168				
74	243	131	228	18	112	130	144				
91	143	236	34	41	185	204	92				
191	216	186	14	110	77	8	35				
188	27	103	137	182	64	59	105				
215	247	238	126	138	26	227	55				
21	84	104	51	135	113	255	172				
56	89	187	28	62	32	45	65				
36	251	152	116	189	7	108	46				
202	162	159	83	31	154	11	231				
106	13	0	217	157	229	102	118				
82	85	176	97	214	151	6	4				
142	245	134	60	225	165	3	39				
86	101	90	127	197	72	117	146				
47	195	42	128	100	253	174	209				
25	239	114	219	244	234	163	190				
183	235	54	98	153	121	123	38				
40	180	179	139	203	70	5	24				
43	199	224	213	210	220	173	241				
23	88	196	79	242	58	9	73				
141	160	193	181	19	233	63	80				
30	81	111	226	175	150	207	222				
17	119	230	96	71	87	133	132				
95	169	155	212	66	49	205	2				
76	115	37	194	57	22	223	178				
16	12	93	237	240	33	206	69				
53	158	148	15	122	136	161	246				
201	44	171	67	184	109	252	50				
170	145	149	140	94	218	156	208				
1	129	68	48	254	164	250	167				
248	125	177	166	232	120	107	99				
249	221	52	124	10	211	61	147				

```
def next_byte():
    i := (i+1) mod 256
    j := (j+S[i]) mod 256
    swap(S[i], S[j])
    return S[(S[i] + S[j]) mod 256]

next_byte() = 119
```

Historical PRNG: RC4

i = 4 j = 20								
75	78	198	20	41	200	29	168	
74	243	131	228	18	112	130	144	
91	143	236	34	192	185	204	92	
191	216	186	14	110	77	8	35	
188	27	103	137	182	64	59	105	
215	247	238	126	138	26	227	55	
21	84	104	51	135	113	255	172	
56	89	187	28	62	32	45	65	
36	251	152	116	189	7	108	46	
202	162	159	83	31	154	11	231	
106	13	0	217	157	229	102	118	
82	85	176	97	214	151	6	4	
142	245	134	60	225	165	3	39	
86	101	90	127	197	72	117	146	
47	195	42	128	100	253	174	209	
25	239	114	219	244	234	163	190	
183	235	54	98	153	121	123	38	
40	180	179	139	203	70	5	24	
43	199	224	213	210	220	173	241	
23	88	196	79	242	58	9	73	
141	160	193	181	19	233	63	80	
30	81	111	226	175	150	207	222	
17	119	230	96	71	87	133	132	
95	169	155	212	66	49	205	2	
76	115	37	194	57	22	223	178	
16	12	93	237	240	33	206	69	
53	158	148	15	122	136	161	246	
201	44	171	67	184	109	252	50	
170	145	149	140	94	218	156	208	
1	129	68	48	254	164	250	167	
248	125	177	166	232	120	107	99	
249	221	52	124	10	211	61	147	

```
def next_byte():
    i := (i+1) mod 256
    j := (j+S[i]) mod 256
    swap(S[i], S[j])
    return S[(S[i] + S[j]) mod 256]

next_byte() = 129
```

Historical PRNG: RC4

i = 4 j = 20									
75	78	198	20	41	200	29	168		
74	243	131	228	18	112	130	144		
91	143	236	34	192	185	204	92		
191	216	186	14	110	77	8	35		
188	27	103	137	182	64	59	105		
215	247	238	126	138	26	227	55		
21	84	104	51	135	113	255	172		
56	89	187	28	62	32	45	65		
36	251	152	116	189	7	108	46		
202	162	159	83	31	154	11	231		
106	13	0	217	157	229	102	118		
82	85	176	97	214	151	6	4		
142	245	134	60	225	165	3	39		
86	101	90	127	197	72	117	146		
47	195	42	128	100	253	174	209		
25	239	114	219	244	234	163	190		
183	235	54	98	153	121	123	38		
40	180	179	139	203	70	5	24		
43	199	224	213	210	220	173	241		
23	88	196	79	242	58	9	73		
141	160	193	181	19	233	63	80		
30	81	111	226	175	150	207	222		
17	119	230	96	71	87	133	132		
95	169	155	212	66	49	205	2		
76	115	37	194	57	22	223	178		
16	12	93	237	240	33	206	69		
53	158	148	15	122	136	161	246		
201	44	171	67	184	109	252	50		
170	145	149	140	94	218	156	208		
1	129	68	48	254	164	250	167		
248	125	177	166	232	120	107	99		
249	221	52	124	10	211	61	147		

Vulnerabilities include

- first few bytes expose key
- bias in consecutive pairs
- bias toward repeating pairs

Modern PRNG: ChaCha

chacha(*key, counter, nonce*) → PRNG

"expa"	"nd 3"	"2-by"	"te k"
key	key	key	key
key	key	key	key
counter	counter	nonce	nonce

<https://en.wikipedia.org/wiki/Salsa20>

Applied alternately to columns then diagonals:

```
quarter_round(a, b, c, d):  
    a += b; d ⊕= a; d <<<= 16  
    c += d; b ⊕= c; b <<<= 12  
    a += b; d ⊕= a; d <<<= 8  
    c += d; b ⊕= c; b <<<= 7
```

Summary

Stream ciphers use a PRNG plus \oplus

Don't reuse a single key

Confidentiality only; need more layers for other goals

Modern CSPRNGs use a mixture of <<< and \oplus on internal state